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Thank you!

Aging and Longevity:

Mortality laws and mortality forecasts for aging populations

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Abstract

Increasing numbers of people surviving to advanced ages pose serious challenge to government pension systems and to the most industrialized societies. In the majority of developed countries fertility already reached very low levels and has little chances of radical changes, so mortality and mortality at advanced ages in particular is the main driving force behind the future population changes. Therefore, accurate estimates of mortality at advanced ages are essential to improving forecasts of mortality and the population size of the oldest old age group. In this article we present some new approaches to mortality and population projections at older ages. We apply modified method of mortality shifting to population of Sweden and make mortality projections up to 2070. Specifically, we identify the best time interval for identifying the rate of mortality decline to use in mortality extrapolation. In the case of Sweden, the best interval is 1980 through 2008 years for both men and women. For men, the rate of mortality decline was almost twice as high as the rate for female. Finding the best formula for extrapolating mortality for ages beyond 100-105 years is another important issue in mortality projections given increasing longevity in industrialized countries. Study of several single-year extinct U.S. birth cohorts found that mortality trajectory at advanced ages follows the Gompertz law up to the ages 106 years without significant mortality deceleration. These findings are supported by another study of independent data on siblings of centenarians drawn from verified and accurate U.S. family histories. Using these two simple assumptions (log-linear decline of mortality over time and Gompertz law working at advanced ages), we made mortality projections for Swedish males and females for the next 60 years. According to these projections, life expectancy at age 25 will increase from 54.07 in 2005 to 62.71 in 2050 for men and from 58.20 to 63.50 for women. If this tendency of mortality decline continues then in 2059 life expectancy at age 25 for men may surpass that of women. These advances in life expectancy will not result in population growth and in the absence of migration the 'native' population of Sweden is expected to decline after 2036 (assuming unchanged birth rate).

Introduction

Population aging is a global phenomenon, which is particularly expressed in industrialized countries. The proportion of older people in these countries grows now with accelerated pace mainly due to increasing longevity, because fertility there already reached very low levels.

Population aging is expected to continue over the next few decades, eventually leading to the global convergence in the proportion of older people. Although fertility decline was the main cause of population aging in the past, the process of population aging in contemporary societies is determined by declining mortality at older ages. Thus, mortality studies and projections for older ages are particularly important for making accurate demographic forecasts of population aging.

This paper is focused on mortality changes at older ages because these changes are now the main driving force behind both increases in life expectancy and population aging. In this article we present some new approaches to mortality forecasts and population projections at older ages.

Using parametric models (mortality laws) for mortality projections

Parametric models of mortality represent a useful tool in demographic and actuarial projections of mortality. One of the first and most successful attempts to express the dependency between mortality and age mathematically was that of the English actuary Benjamin Gompertz, dating to 1825 (Gompertz 1825):

$$\mu_x = R_0 \exp(\alpha x)$$

where μ_x is the force of mortality (hazard rate) at age x; and α and R_0 are the parameters of the equation. This formula, which describes the mortality of people older than 20, was called the Gompertz law, and its parameters were named the Gompertz parameters. Subsequently, the Gompertz law began to be used widely for describing the mortality of laboratory animals (Gavrilov and Gavrilova 1991).

In his work, Gompertz noted that, in addition to the mortality which grows exponentially with age, there can also exist a component of mortality which is independent of age: "*It is possible that death may be the consequence of two generally coexisting causes: the one chance, without previous disposition to death or deterioration, or increased inability to withstand destruction*" (Gompertz 1825). However, for the analysis of the life tables which were then available, Gompertz considered it possible to restrict himself solely to the exponential component of mortality. Not until 35 years later, in 1860, did another actuary William Makeham add the age-independent component to the Gompertz formula (Makeham 1860). This component, usually denoted by the letter A, received the name of the Makeham parameter (Gavrilov and Gavrilova 1991). Thus the formula appeared which we now know as the Gompertz-Makeham law:

$$\mu_x = A + R_0 \exp(\alpha x)$$

A is the age-independent component of mortality, which we called the background component of mortality in analogy with background radiation (Gavrilov and Gavrilova 1979, 1991); The second term of this equation is the age-dependent component of mortality (Gavrilov and Gavrilova 1979), which is called now the senescent component of mortality (Bongaarts 2005). As

can be seen from what has been said, the age-dependent component of mortality is an exponential. In the particular case in which the background mortality can be ignored (for example under good laboratory conditions or in contemporary industrialized countries), the total force of mortality grows exponentially with age, i.e. in accordance with the Gompertz law.

The Gompertz-Makeham formula describes the life span distributions of a wide variety of biological species (drosophila, mosquitoes, flour beetles, mice, rats, horses and mountain sheep), including humans (Gavrilov and Gavrilova 1991). There are some reports that the competing Weibull formula (power law) fits data better than the Gompertz formula. These reports are usually based on analysis of a few life tables for populations of small size (often less than 100 animals). Our comparative study of Weibull and Gompertz models using data for 260 human life tables and 15 life tables for fruit flies (with initial population size of 1000 or more animals) demonstrated that on average the Gompertz-Makeham law fits mortality at adult ages better than the Weibull-Makeham law (Gavrilov and Gavrilova 1991) although in some rare cases Weibull formula shows better fit of mortality data.

Subsequently, there were many attempts to modify the Gompertz law. The most common way of modifying the Gompertz function is to use what are called logistic equations. The earliest such formula was proposed by Perks and the latest and the most widely used one was proposed by Kannisto and is called a Kannisto formula (Kannisto 1994):

$$\mu_x = \frac{B\exp(\alpha x)}{1 + B\exp(\alpha x)}$$

The formulas listed above are applicable to mortality of adult population (usually above age 20 years). There were also attempts to describe mortality in the entire age interval, such as Heligman-Pollard (Heligman and Pollard 1980) and Siler (Siler 1979) formulas.

Parametric formulas can be used in population projections by analyzing historical trends of their parameters. For example, in 1979, during an analysis of the historical changes in the mortality of the Swedish male population, it was found that the age-dependent component of mortality in the Gompertz-Makeham formula demonstrates surprising historical stability despite rapid decline in age-independent mortality (Makeham term) (Gavrilov and Gavrilova 1979). Further more careful investigation confirmed the validity of this phenomenon (Gavrilov and Gavrilova 1991; Gavrilov, Gavrilova and Nosov 1983) and the study of historical time series of mortality for 17 countries permitted the conclusion that it was quite general in character (Gavrilov and Gavrilova 1991). Figure 1 shows changes in total, background and senescent mortality for Swedish males. It can be seen that the background component of mortality is the only mortality component, which has significantly changed over the studied period (1900-1970). The senescent mortality (and two Gompertz parameters), turn out to be practically unchanged, despite the sharp fall in total mortality in the 20th century. We observe that the substantial decline in mortality rates in Sweden at the beginning of the 20th century can be explained by a decrease in the Makeham component while the Gompertz component remained virtually constant during the same period. In the 1960s, as the Makeham component had almost reached zero, it became foreseeable that the rapid decline in mortality rates would come to an end. And this is what happened in fact in the 1960s (Gavrilov et al. 1983). Thus, based on the observation of the mortality tables for the first half of the 20th century, it was possible to predict a "biological limit" to the force of mortality. For example, at the beginning of the 20th century, total mortality was substantially higher in Norway than in Denmark. However,

based on the observation that the Gompertz component was considerably lower in Norway, we were able to predict a reversal in the trend as the Makeham component declines. This is exactly what happened (Gavrilov and Gavrilova 1991). Similarly, in Italy, the mortality rates of men and women were virtually identical at the beginning of the 20th century but the biological limit for women was lower due to the lower corresponding Gompertz component. Based on data from the beginning of the 20th century, we were able to predict that eventually, women's mortality would become lower than male mortality (Gavrilov and Gavrilova 1991). However after the 1960s, new unexpected trends in mortality have started. These trends were not well visible at the time of this study, although some indications of further mortality decline have been already noticed (Gavrilov and Gavrilova 1991; Gavrilov and Nosov 1985).

Recently Bongaarts developed further the method based on studying historical trends of Gompertz-Makeham parameters suggesting use of logistic formula for mortality forecasting (Bongaarts 2005). This modification was reasonable because mortality rates for period life tables in Human Mortality Database used in his study were fitted by logistic formula after age of 85 years (Wilmoth et al. 2007). His study analyzed historical trends in the interval 1950-2000 years for 14 countries and confirmed decline of the background mortality and stability of the slope parameter in the Gompertz term found in the previous studies. Due to the limited number of life tables (no life tables before 1950 were used) this study could not demonstrate the full scale of decline in background mortality during the first half of the 20th century. However this study revealed another interesting regularity that could not be fully analyzed in the past: decline in the pre-exponential multiplier of senescent mortality. Decline of this parameter (called the level parameter by Bongaarts) in conjunction with stability of the slope parameter in the Gompertz term means that the senescent component of mortality in developed countries undergoes parallel shift in semi-log coordinates over time. This pattern of mortality change was called a shifting logistic model (Bongaarts 2005). Based on this mortality pattern, Bongaarts suggested a new approach to mortality projections. This approach is based on estimating parameters of the logistic formula for a number of years and extrapolating the values of three parameters (background mortality and two parameters of senescent mortality) to the future (Bongaarts 2005, 2009).

Figure 2 illustrates the two phases of mortality decline in the 20th century. When we analyze mortality data on a larger time scale, we observe a decline of mortality in all age groups between 1925 and 1955 except for the elderly where death rates remained relatively constant. Note that mortality trajectories for 1925 and 1955 are close to each other at older ages, which corresponds to the stability of the senescent mortality during the first half of the 20th century. After the 1950s, the parallel shift of mortality has been observed, which corresponds to the model proposed by Bongaarts (for example, compare mortality trajectories for 1955 and 2005).

It should be noted that in addition to the approach based on the background and senescent mortality there is another way of mortality partitioning. In 1952, Jean Bourgeois-Pichat attempted to predict population mortality using the idea of endogenous and exogenous causes of death (Bourgeois-Pichat 1952). In the exogenous causes of death he included infectious and parasitic diseases, respiratory diseases, accidents, poisonings, and violence. The endogenous causes of death included malignant neoplasms, circulatory diseases, and the remaining causes of death (Bourgeois-Pichat 1952). In Bourgeois-Pichat's opinion, the evolution of human mortality can be likened "to the erosion of soil composed of two kinds of rock: soft rock and hard rock." At first, the "soft rock" is quickly eroded (exogenous causes of death), then the "hard rock" slowly erodes (endogenous causes of death). On the basis of these ideas, it was predicted that medical advances in eliminating the exogenous causes of death would lead to the endogenous causes of death coming to the fore. Thus, Bourgeois-Pichat formulated "the concept of a temporary limit on mortality decline", and even calculated the level of this "temporary limit" for each age, calling it "the biological limit of mortality decline" (Bourgeois-Pichat 1952, 1979). Later Carnes and Olshansky

developed this approach further suggesting so-called biologically motivated partitioning of mortality into extrinsic and intrinsic mortality based on cause-of-death information (Carnes and Olshansky 1997). Information on causes of death is used sometimes in actuarial practice for making mortality projections although these projections usually underestimate future mortality decline. Mortality partitioning proved to be a useful tool for mortality projections in the past when background mortality (analog of extrinsic mortality) was high, but this approach is less useful now when background mortality is close to zero and does not change significantly over time. In addition to that, in some cases, it is simply impossible to establish whether death is exogenous or endogenous. For example, a patient may be suffering from several diseases, none of which alone would lead to death, but which are lethal in combination. So this approach has a limited applicability in demographic practice now.

Despite the usefulness of parametric approach to mortality projections it has serious limitations. The main limitation is a dependence on the particular formula, which makes this approach too rigid for responding to possible changes in mortality trends and fluctuations. In the next section we consider some methods of mortality projections based on non-parametric approaches.

Nonparametric approach to mortality projections

The Lee-Carter method is now one of the most widely used methods of mortality projections in demography and actuarial science (Lee and Miller 2001; Lee and Carter 1992). Its success is stemmed from the shifting model of mortality decline observed for industrialized countries during the last 30-50 years. The Lee-Carter method is applied to the logarithm of mortality and is based on the following formula for hazard rate (or central death rate) (Lee 2000):

$$\ln(\mu_{x,t}) = \mathrm{a}(x) + b(x)k(t)$$

where *a(x)*, *b(x)* and *k(t)* are parameters to be estimated. This model does not produce a unique solution and Lee and Carter suggested applying the following constraints (Lee 2000; Lee and Carter 1992):

$$\sum_t \mathrm{k}(t) = 0; \quad \sum_x \mathrm{b}(x) = 1$$

The first constraint implies that the parameter a(x) is an empirical average of the logarithm of mortality at age x over time. In the first stage of the Lee-Carter method, coefficients a(x) and b(x) are estimated. In the second stage, the empirical values of k(t) coefficients are estimated using the following formula (Lee 2000):

$$D_t = \sum \exp(a_x + b_x k_t) N_{x,t}$$

where D_t is total number of deaths in year t, and N_{xt} is the population aged x in year t.

The empirical time series of *k* coefficients can be extrapolated from the base period to the future which is essentially a linear extrapolation.

Note that the Lee-Carter method is modeling logarithms rather than absolute values of mortality and hence is based on multiplicative model of mortality change over time (rather than additive one as in the case of Gompertz-Makeham model). On the other hand, the Lee-Carter model is not based on any particular parametric formula and allows researchers to make a compact description of a large set of mortality data without excessive loss of information. In contrast to aggregated indicators such as life expectancy, the knowledge of the Lee-Carter model parameters allows researchers to reconstruct values of age-specific mortality rates and their temporal evolution with reasonable accuracy. One limitation of this method is related to the assumption that historical evolution of mortality at all age groups is driven by one factor only (parameter b) (Lee 2000). However a factor analysis of mortality evolution (see Annex for details) found that this approach turns out to be overly simplistic (Gavrilov and Gavrilova 1991; Gavrilov and Nosov 1985). For example, factor analysis of mortality dynamics over the period of 1900-2007 in developed countries found that at least two time-dependent factors are responsible for observed decline of mortality (younger age groups have a different factor of mortality decline compared to older groups). One-factor model could be applicable to earlier historical periods only (before 1950s), when a decline in mortality rates was driven mainly by a decrease of the background mortality (the Makeham parameter of the Gompertz-Makeham law) (Gavrilov and Gavrilova 1991; Gavrilov et al. 1983). It is obvious that the Lee-Carter model is not well applicable to mortality modeling during the period 1900-1950 because of additive rather than multiplicative model of mortality decline during this time.

In order to overcome limitation of one-factor model of mortality and to determine the true number of factors underlying mortality changes over time, we conducted a factor analysis of mortality for Swedish data over the period of 1900-2008 (see Annex 1 for more detail). We used so-called P-technique of factor analysis when the analysis occurs across different time points or observations (values of hazard rates at different years) for ages 25 through 85 (Uberla 1977). We applied factor analysis procedure with promax rotation method using the Stata, release 11 statistical package. Data on men and women were analyzed separately. We identified two factors capable of explaining almost 98% of the variance in the temporal changes of hazard rates. Thus, for more accurate description of mortality evolution, the following model would be preferable:

$$\mu(x, t) = a_0(x) + a_1(x)F_1(t) + a_2(x)F_2(t)$$

where x is age, t is time, $\alpha(x)$, $\alpha_1(x)$, $\alpha_2(x)$ are three sets of parameters depending on age only, while $F_1(t)$ and $F_2(t)$ are two sets of parameters depending on time only (sets of coefficients determined by factor analysis models).

By studying the variation of these factors over time, we noted that the first factor - comparable to the Makeham component and observed in the "young ages" population (see Table 3 in the Annex) - declined from the beginning of the century. The second factor - comparable to the senescent mortality and chiefly concerning the "old ages" population - remained remarkably stable over a period of 1900-1950 (see Figurers 3 and 4). Without more recent data, we might predict continued historical stability of this factor. However, a radical change has occurred after the 1950s and mortality has begun to decline among older people while the mortality of the younger age groups has already reached very low levels close to zero. Thus, factor analysis of the time series of mortality confirms the preferential reduction in the mortality of old-aged people in the recent years. Also note that for males, the senescent factor started its rapid decline significantly later compared to females.

Observations made before 1950 enrich the historical data as a whole but they are liable to distort the results of mortality projections. However, for the future forecasts it is better to use more

recent data, which take account of the change in the patterns of mortality decline. What conclusions may be drawn at this point? In the past, it was possible to argue that there was a biological limit underlying the observed mortality rate. We have observed, however, that these limits can be pushed back thanks to technological or medical progress. Although it is equally impossible to conclude that the mortality force is tending toward zero, the short-term trend is clearly oriented downwards.

The approach based on the factor analysis has several advantages. First it is able to determine the number of factors affecting mortality changes over time. Second, this approach allows researchers to determine the time interval, in which underlying factors remain stable or undergo rapid changes. For example, Figures 3 and 4 clearly demonstrate that the second factor was relatively stable in the past but now is rapidly declining for both men and women. However, this rapid decline started almost 30 years later in men compared to women. Most methods of mortality projections are not able to identify the best base period of time for mortality changes that should be extrapolated in the future. For example, the Lee-Carter method suggests using the longest possible interval. It is clear that such approach will not bring the most accurate mortality forecast. Using the results of factor analysis we may conclude that 1980 is the best starting year for mortality projections and 1980-2008 is a useful base period for mortality extrapolation. After 1980, the senescent factor demonstrates a stable linear decline and it is reasonable to suggest that this decline will continue into the foreseeable future.

Taking into account the shifting model of mortality change it is reasonable to conclude that mortality after 1980 can be modeled by the following log-linear model with similar slope for all adult age groups:

$$\ln(\mu_{x,t}) = a(x) - kt$$

Figure 5 illustrates the validity of suggested model for Swedish men. Note that the logarithm of mortality declines linearly in all observed age groups with the same linear slope. Similar regularity is observed for Swedish women.

We suggest here to use the shifting model of mortality with the slope parameter based on mortality rate of change after 1980. So far we analyzed mortality in the age interval 25-85 years. However, there is a question related to the mortality pattern at advanced ages. Bongaarts used logistic formula for mortality modeling and this formula is now the most popular way of mortality modeling at advanced ages. Should we use this formula for mortality projections at advanced ages? We attempt to answer this question in the next section.

Mortality trajectories at very advanced ages

It is now considered as an established fact that mortality at advanced ages has a tendency to deviate from the Gompertz law, so that the logistic model often is used to fit human mortality (Horiuchi and Wilmoth 1998). The estimates of hazard rate at extreme ages are difficult to obtain because of small numbers of survivors to these ages in most countries. Data for extremely long-lived individuals are scarce and subjected to age exaggeration. Traditional demographic estimates of mortality based on period data encounter well known denominator problem. More accurate estimates are obtained using the method of extinct generations (Vincent 1951). In order to obtain good quality estimates of mortality at advanced ages researches are forced to pool data for several calendar periods. Single-year life tables for many countries have very small numbers of survivors to age 100 that makes estimates of mortality at advanced ages unreliable. The aggregation of

deaths for several calendar periods however creates a heterogeneous mixture of cases from different birth cohorts. Mortality deceleration observed in these data might be an artifact of data heterogeneity. In addition to that, many assumptions about distribution of deaths in the age/time interval used for mortality estimation are not valid at extreme old ages when mortality is particularly high.

Mortality deceleration and subsequent mortality plateau (logistic formula) is often presented as universal mortality law. Indeed, the existence of mortality plateaus is well established for a number of lower organisms, mostly insects, including fruit flies , medflies and house flies (Carey et al. 1992; Curtsinger et al. 1992; Curtsinger, Gavrilova and Gavrilov 2006; Gavrilov and Gavrilova 2006; Vaupel et al. 1998). In the case of mammals, however, data are much more controversial. Although Lindop and Sacher reported short-term periods of mortality deceleration in mice at advanced ages (Lindop 1961; Sacher 1966) Austad later argued that rodents do not demonstrate mortality deceleration even in the case of large samples (Austad 2001). Study of baboons found no mortality deceleration at advanced ages (Bronikowski et al. 2002). Recent study of mortality in primates also failed to find mortality deceleration at older ages (Bronikowski et al. 2011). In the case of humans, this problem is not yet resolved, because of scarceness of data and/or their low reliability. Thus, more studies on larger human birth cohorts are required to establish with certainty the true mortality trajectory at advanced ages.

We have carried out a study based on the analyses of data taken from the U.S. Social Security Administration Death Master File (SSA DMF). Social Security Administration Death Master File (DMF) is a publicly available data source that allows a search for deceased individuals in the United States using various search criteria: birth date, death date, first and last names, social security number, place of last residence, etc. This resource covers deaths that occurred in the period 1937-2010 and captures about 95% of deaths recorded by the National Death Index (Sesso, Paffenbarger and Lee 2000). According to other estimates, DMF covers about 92-96 percent of deaths for persons older than 65 years (Hill, Rosenwaike, 2001).

Social Security Administration Death Master File (DMF) was used in our study of age-related mortality dynamics after ages 88 years. The advantage of this data source is that some already extinct birth cohorts covered by DMF could be studied by the method of extinct generations (Kannisto 1988, 1994; Vincent 1951). Information available in DMF includes: names of the deceased, his/her social security number, date, month, year of birth, month and year of death, state of SSN issuance, place of the last residence. In this study information from the DMF was collected for individuals who lived 88 years and over and died before 2011. DMF database is unique because it represents mortality experience for very large birth cohorts of the oldest-old persons. In this study mortality measurements were made for cohorts, which are more homogeneous in respect to the year of birth and historical life course experiences. Availability of month of birth and month of death information provides a unique opportunity to obtain hazard rate estimates for every month of age, which is important given extremely high mortality after age 100 years. Despite certain limitations, this data source allows researchers to obtain detailed estimates of mortality at advanced ages. We already used this data resource for centenarians' age validation in the study of centenarian family histories (Gavrilova and Gavrilov 2007). This data resource is also useful in mortality estimates for several extinct or almost extinct birth cohorts in the United States.

We obtained data for persons who died before 2011 and were born in 1875-1895. Assuming that the number of living persons belonging to these birth cohorts in 2010 is close to zero, it is possible to construct a cohort life table using the method of extinct generations, which is considered to be the most accurate method to study old-age mortality (Kannisto 1994). In the first stage of our analyses we calculated an individual life span in completed months:

Lifespan in months = (death year – birth year) x 12 + death month – birth month

Having this information it is possible to estimate hazard rates at each month of age by standard methods of survival analysis. All calculations were done using the Stata statistical software, release 11 (StataCorp 2009). This software calculates nonparametric estimates of major survival functions including the Nelson-Aalen estimator of hazard rate (force of mortality). In this study, survival times were measured in months, so the estimates of hazard rates initially had a dimension of $month^{-1}$. For the purpose of comparability with other published studies, which typically use the $year^{-1}$ time scale, we transformed the monthly hazard rates to the more conventional units of $year^{-1}$, by multiplying these estimates by a factor of 12 (one month in the denominator of hazard rate formula is equal to 1/12 year). It should be noted that hazard rate, in contrast to probability of death, can be greater than 1, and therefore its logarithm can be greater than 0 (and we indeed observed these values at extreme old ages in some cases). In this paper we focus our analyses on 1886-1895 birth cohorts, because we found that data quality for earlier cohorts is not particularly good.

Results of the hazard rate estimates for 1891 birth cohort are presented in Figure 6. Note that mortality trajectory in semi-log scale is linear up to the age 105-106 years. After age 106 years data points show very high variation suggesting declining data quality (possible age misreporting). One approach to evaluate data quality at advanced ages is to calculate female to male ratio at advanced ages. Taking into account that female mortality is always lower than male mortality it is reasonable to expect that the female-to-male ratio should continuously increase with age. On the other hand, old men have a tendency for age exaggeration and in populations with poor age registration there is a relative excess of men at very advanced ages (Caselli et al. 2006; Willcox et al. 2008). We calculated female-to-male ratio after age 95 years for 1887-1892 U.S. birth cohorts from the SSA DMF. Figure 7 demonstrates the age dependency of this ratio for pooled sample of 1887-1892 birth cohorts (these cohorts have similar levels of mortality). Note that the female to male ratio is growing steadily with age up to ages 106-107 years. After this age the female-to-male ratio starts to decrease indicating declining quality of age reporting. Thus, the estimates of hazard rates obtained from the SSA DMF are of acceptable quality up to the age of 106 years. For this reason we used age interval 88-106 years for mortality modeling.

Next step of our study was to compare two competing models of mortality at advanced ages - the Gompertz and the logistic models - using data of reasonably good quality. Study of data quality of at advanced ages described above suggests that age reporting among the oldest-old in the United States is good until the age of 106 years. It means that comparing mortality models beyond this age is not feasible because of poor quality of mortality data. It was shown that age reporting for persons applied to Social Security Numbers in the Southern states of the U.S. is significantly less accurate compared to persons applied in the Northern states regardless of race. (Rosenwaike and Stone 2003). For this reason, we used a subsample of deaths for persons applied to SSNs in the 'Northern' states and born in 1886-1895, because these data have reasonably good quality. We applied the Gompertz and logistic (Kannisto) models (Thatcher, Kannisto and Vaupel 1998) to mortality modeling in the age interval 88-106 years using nonlinear regression method for parameter estimation. Calculations were performed using Stata statistical software, release 11 (StataCorp 2009). Bayesian information criterion (BIC) was used as a goodness-of-fit measure. Table 1 shows values of BIC for both Gompertz and logistic model for ten studied birth cohorts. Note that in 8 out of 10 cases (studied birth cohorts), the Gompertz model demonstrates better fit (lower BIC) than the logistic model for age interval 88-106 years.

At this moment, we cannot make a conclusion that Gompertz model fits mortality data better than the logistic model beyond the age of 106 years, because of low quality of age reporting at very old ages. At the same time, the data indicate that the Gompertz model fits mortality data well until

the age 106 years. Taking into account that survival beyond age 106 years is rather rare event, it would be reasonable to suggest the use of Gompertz model rather than logistic model for closing cohort life tables in demographic practice. In this case, mortality modeling could be done first for hazard rate (mortality force) function and then all life table functions (including probability of death, q_x) could be derived on the basis of modeled values of hazard rate. Thus the Gompertz model fits mortality reasonably well up to the age of 106 years. Comparison of DMF data with published 1900 U.S. actuarial cohort life table (Bell, Wade and Goss 1992) showed that DMF mortality estimates are similar to mortality estimates obtained from the 1900 cohort life table. The maximum likelihood estimator of the Gompertz slope parameter for mortality in 1894 cohort measured in the interval 88-106 years for DMF data (0.0786 $year^{-1}$, 95%CI: 0.0786-0.0787) does not differ from the slope parameter calculated over the age interval 40-104 years in 1900 life table: 0.0785 $year^{-1}$, 95%CI: 0.0772-0.0797.

Our study of late-life mortality based on the data from the U.S. SSA Death Master File suggests that for rather homogeneous single-year birth cohorts mortality at advance ages does not decelerate up to very advanced ages. In order to make an independent check of our findings we used another dataset. We have developed and analyzed a new computerized database on 1,711 validated centenarians born in the United States in 1880-1895, as well as their 13,392 shorter-lived siblings. These data were collected from the Rootsweb publicly available database using web-automation technique for centenarians having information on lifespan of their parents and the majority of their siblings. Additional validation of centenarian age through the SSA DMF and early censuses ensured high quality of life histories and information on siblings in particular. For the purpose of mortality study, we used only those siblings who were born before 1880, i.e. not in the same time window as the selected centenarians. As a result, 1,895 siblings born in 1856-1879 were identified and 1681 siblings survived to age 60 were used for mortality analysis. Figure 8 shows the hazard rate trajectory (in semi-log scale) for this group of siblings using 6-month age intervals for hazard rate estimation by actuarial method (Kimball 1960). Note that mortality trajectory after age 60 years does not show a tendency for deceleration despite rather heterogeneous nature of the sample (mixture of different birth cohorts, men and women). This example suggests that mortality deceleration is not a universal phenomenon at advanced ages, but rather a result of age misreporting, data heterogeneity and problems with proper estimation of hazard rates.

Few people survive to advanced ages and, in standard mortality tables, it is frequently necessary to compile data over an entire decade to obtain a sufficiently large sample. Our work shows that the observed deceleration in measured mortality rates could result in part from the heterogeneity of the data. There consequently remains a great deal of research to carry out if we are to improve our understanding of mortality at advanced ages. The second problem we examined is frequently overlooked by demographers and actuaries: the problem of correct estimation of the instantaneous mortality rate (hazard rate). At the most advanced ages, the rates of death are so high that it is impossible to assume that the number of dying is distributed uniformly within the studied one-year intervals. As a result, the estimates of mortality rates (or central death rates) are biased downwards at advanced ages. And finally, the third problem is related to the fact that elderly people tend to round their ages up, thereby exaggerating their true age. In the United States, this may have made impaired the accuracy of mortality rate estimates in the past.

Making mortality forecasts using the information from the observed phenomena

In the previous sections we already demonstrated that the classic Gompertz model can be used for mortality modeling up to very old ages. Extending this model to age 106 years is sufficient for most countries to close life table because only few individuals survive to these ages even in the countries with low mortality. The factor analysis of mortality changes indicates that age-dependent (senescent) mortality continues to decline and this decline does not demonstrate any indications of slowing down. This observation means that the traditional Lee-Carter approach can be used for mortality forecasting. The study of mortality changes using factor analysis demonstrates that mortality trends after 1980 can be continued beyond 2011 for both men and women. As can be seen from Figure 5, logarithms of mortality after 1980 for Swedish men show practically linear decline over time with similar slope for the studied ages. We can use this property to model mortality decline after 2010 assuming the same rate of mortality changes (in a log scale) for different ages. Changes of mortality after 1980 for Swedish females reveals similar phenomenon although women demonstrate slower rate of mortality decline.

Based on the observed mortality trends estimated that mortality of Swedish males declines by 2 percent per year on average while mortality of Swedish female declines by 1 percent per year. We assumed that these rates of mortality decline for each gender remain unchanged over time and are the same for all age groups. These gender differences in the rate of mortality decline over the last three decades are apparently responsible for the decreasing gender gap in life expectancy observed recently in the majority of developed countries (Glei and Horiuchi 2007).

Additionally, we assume that mortality continues to follow the Gompertz law at advanced ages as shown in the previous section. At the same time, we do not attempt to close life table at any predetermined age (say, 110 years) as it is often made in demographic forecasts. In our projections, the number of the last death is shifting to higher ages as long as mortality continues to decline. Using all the listed assumptions we conducted mortality projections for the next 50-60 years.

We used traditional cohort-component method (Preston, Heuveline and Guillot 2001) to make population projections for Sweden until 2060 assuming unchanged age-specific fertility schedule and no migration (to evaluate changes in mortality on population growth and aging). For our life expectancy and population projections, we used official 2005 Swedish data on population age-sex distribution, 2005 age-specific fertility rates and 2005 life table. Our projected values of life expectancy using this approach are more optimistic than forecasts of the majority of demographers made so far (Waldron 2005). According to our forecasts, life expectancy at birth may reach 90 years in 2070. Another difference between our results and the existing projections is the declining gender gap in life expectancy. It is projected that in 2059 life expectancy of men may outrun that of women if current trends in mortality continue to the future. Figure 9 shows the projected trends in life expectancy at age 25 for Swedish men and women together with the observed values of life expectancy taken from the Human Mortality Database. Note that our method shows good correspondence with real data for male mortality while for females it overestimates the observed life expectancy. Overestimation of life expectancy for women suggests that the selected pace of mortality decline (1 percent per year) is slower for age groups not considered in our analyses (probably age groups over 90). Although we made mortality projections up to year 2070, it is more reasonable to suggest that the observed trend in mortality decline will continue for the next decade with possible uncertain changes after 2020. In this case, the projected life expectancy at birth in 2020 will be 81.53 years for men and 85.16 years for women with possibility of lower value of life expectancy for women. Thus, gender gap in life expectancy will decrease from 4 years in 2006 to 3.5 years in 2020.

These changes in life expectancy will have a profound effect on population aging. Increasing longevity accelerates the pace of population aging. Figure 10 shows the growth of the

proportion of older persons aged 65 and older in the Swedish population. Note that in the nearest future Sweden will experience very rapid population aging, so that the projected number of persons aged 65 and older in 2030 will reach 25% for men and 28% for women (currently 17 and 21 percent). At the same time, the observed gender differences in the degree of population aging will become significantly smaller by 2055.

Conclusions

We demonstrated that the use of factor analysis and simple assumptions about mortality changes over age and time allowed us to provide nontrivial but probably quite realistic mortality forecasts (at least for the nearest future). It is obvious that the proposed approach to mortality forecasting should be country-specific, because each country may demonstrate its own pattern and factor structure of mortality decline over time. However, our preliminary analyses show that the two-factor pattern of mortality decline is observed for the majority of industrialized countries. This study assumes that no changes in mortality patterns are expected in the future. This is most likely an overly simplistic view. Old-age mortality may be affected by different tendencies in the future. On the one hand, an anticipated longevity revolution and new anti-aging technologies are able to slow down the aging process resulting in significant decline of mortality at older ages (Illes, de Grey and Rae 2007). On the other hand, epidemics of obesity and diabetes in developed countries may slow down future mortality decline (Olshansky et al. 2005). These multidirectional trends will shape the pattern of mortality changes in the coming decades and affect population aging in industrialized countries. One important conclusion comes from the mortality projections presented here: these profound future declines of mortality will not result in overpopulation. Without migration and fertility changes, the 'native' population of Sweden will undergo depopulation after 2025. Moreover, it was shown that population changes are surprisingly slow in their response to a dramatic life extension. For example, we applied the cohort-component method of population projections to 2005 Swedish population for several scenarios of life extension and a fertility schedule observed in 2005. Even for very long 100-year projection horizon, with the most radical life extension scenario (assuming no senescence after age 60), the total population increases by 22% only (Gavrilov and Gavrilova 2010). Thus, the future life extension will not significantly increase the total population number although it will significantly accelerate the future population aging.

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ANNEX 1

Application of factor analysis to mortality changes over time.

Factor analysis is a useful statistical method for data compression. Factor analysis originated with the work of Spearman and finds a small number of common latent variables (factors) that linearly reconstruct a larger number of original variables (Kerlinger 1986; Stata Corp. 2009). This approach may be useful for reducing array of mortality rates for a large number of age groups. In this case, the standardized values of mortality rate (or hazard rate) at age x and time t can be presented as follows (Stata Corp. 2009):

$$\mu(x,t) = a_1(x)F_1(t) + a_2(x)F_2(t) + \ldots + a_q(x)F_q(t) + e(x,t)$$

where $a_k(x)$ is the set of linear coefficients called factor loadings, $F_k(t)$ is the kth common factor for observation at time t and e(k,t) is similar to a residual error term. Such model has an infinite number of solutions and various constraints are introduced to make this model determinate. To produce results in a form that is easier to interpret it is necessary to rotate factor matrices (Kerlinger 1986). A rotation, which requires the factors to remain uncorrelated is an orthogonal rotation, while others are oblique rotations (Stata Corp. 2009). In our analyses we applied oblique promax rotation method since it does not impose additional constraints of orthogonality on factors.

The initial data set for factor analysis of mortality in our study is represented by a set of Swedish period life tables taken from the Human Mortality Database (Human Mortality Database 2011). Data were analyzed separately for men and women. Hazard rates for ages 25-85 years were calculated on the basis of age-specific survival numbers (l_x) using Sacher formula (Sacher 1956). Factor analysis was applied to an array of age-specific values of hazard rates for years from 1900 to 2008. We applied so-called p-technique of factors analysis where values of hazard rates at different points of time were considered as observations while hazard rate values at different ages were considered as variables. Table 2 shows eigenvalues and percent of explained mortality variation for the top five factors.

It is a common practice in statistics to leave factors with eigenvalues greater than one for further analyses. Note that the first two factors explain more than 97 percent of historical variation in mortality. Table 3 shows values of rotated factor loadings (for hazard rates at selected ages) for the first two factors in the case of Swedish women. It is clear from the Table 3 that the first factor is a “young-age” factor because of very high factor loadings for mortality rates at ages 25-45 years. On the other hand, the second factor can be called an “old-age” factor because of high factor loadings for mortality rates at ages 65-85 years. As follows from Figure 4, the first factor (corresponding to background mortality) demonstrated rapid decline during the period 1900-1950. Its effect can be illustrated by Figure 2 (mortality curves for 1925 and 1955). Note that the main changes from declining background mortality are observed at younger ages while mortality at older ages remains relatively stable. Decline in the second factor happened after the 1970s (see Figure 4) and the effect of this decline on mortality is clear when mortality data for 1955 and 2005 are compared. Taking into account that Figure 2 show mortality trajectories in semilog scale it is obvious that absolute changes in mortality during 1955-2005 period were the highest at older ages.

Tables:

Table 1. Comparison of goodness-of-fit (Bayesian Information Criterion, BIC) for Gompertz and logistic models of mortality [†.]

Birth cohort	Cohort size at age 88 years, persons	Bayesian Information Criterion (BIC)	
		Gompertz model[††]	Logistic model
1886	111,657	**-594776.2**	-588049.5
1887	114,469	**-625303.0**	-618721.4
1888	128,768	-709620.7	-712575.5
1889	131,778	-710871.1	-715356.6
1890	135,393	**-724731.0**	-722939.6
1891	143,138	**-767138.3**	-739727.6
1892	152,058	**-831555.3**	-810951.8
1893	156,189	**-890022.6**	-862135.9
1894	160,835	**-946219.0**	-905787.1
1895	165,294	**-921650.3**	-863246.6

[†.] Estimates were made in the age interval 88-106 years for ten single-year U.S. birth cohorts and data of enhanced accuracy for individuals applied to SSNs in the Northern states (see explanation in the text).

[††] Cases when the Gompertz model fits data better than the logistic model are highlighted in bold.

Table 2. Results of applying factor analysis† to adult mortality data of Swedish men and women, 1900-2008.

Factor	Eigenvalue	Proportion of variance explained	Cumulative proportion of variance explained
Men			
Factor 1	53.424	0.8766	0.8766
Factor 2	5.888	0.0966	0.9732
Factor 3	0.671	0.0110	0.9842
Factor 4	0.229	0.0038	0.9880
Factor 5	0.079	0.0013	0.9893
Women			
Factor 1	55.356	0.9078	0.9078
Factor 2	4.813	0.0789	0.9867
Factor 3	0.319	0.0052	0.9919
Factor 4	0.119	0.0020	0.9939
Factor 5	0.035	0.0006	0.9944

† Variables included hazard rate values for ages 25-85 years; factor analysis used promax rotation.

Table 3. Rotated factor loadings (for hazard rates at selected ages) for the first two factors after applying factor analysis to historical mortality changes of Swedish women.

Mortality rate at given age, years	Factor loadings for Factor 1	Factor loadings for Factor 2
25	1.0005	-0.0145
35	0.9546	0.0576
45	0.7706	0.2863
55	0.5806	0.4941
65	0.2856	0.7698
75	0.0716	0.9439
85	-0.0080	0.9938

Figures

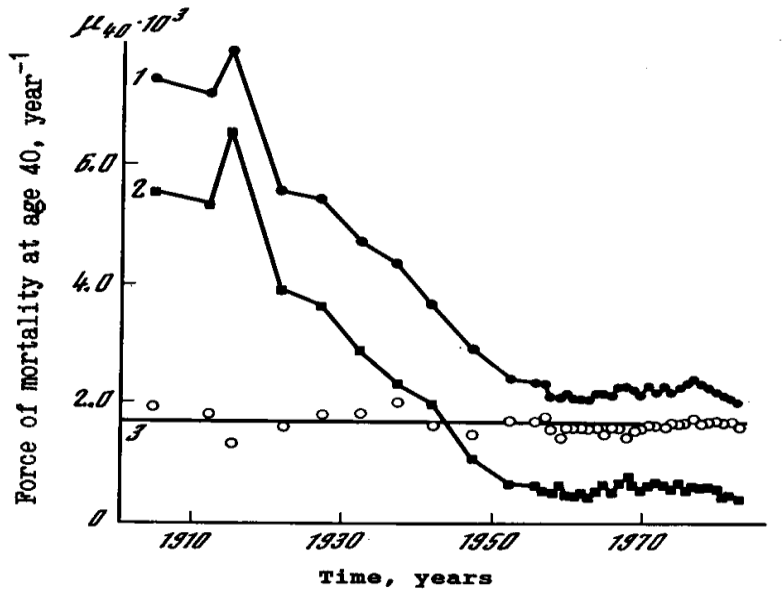

Figure 1. Historical changes of age-independent (background) and age-dependent (senescent) mortality (per 1000) for 40-years old Swedish males. Source: Gavrilov et al., 1983.

1 – total (observed) mortality at age 40 based on official Swedish life tables

2 – background mortality component calculated on the basis of Gompertz-Makeham formula

3 – senescent mortality component at age 40 calculated on the basis of Gompertz-Makeham formula

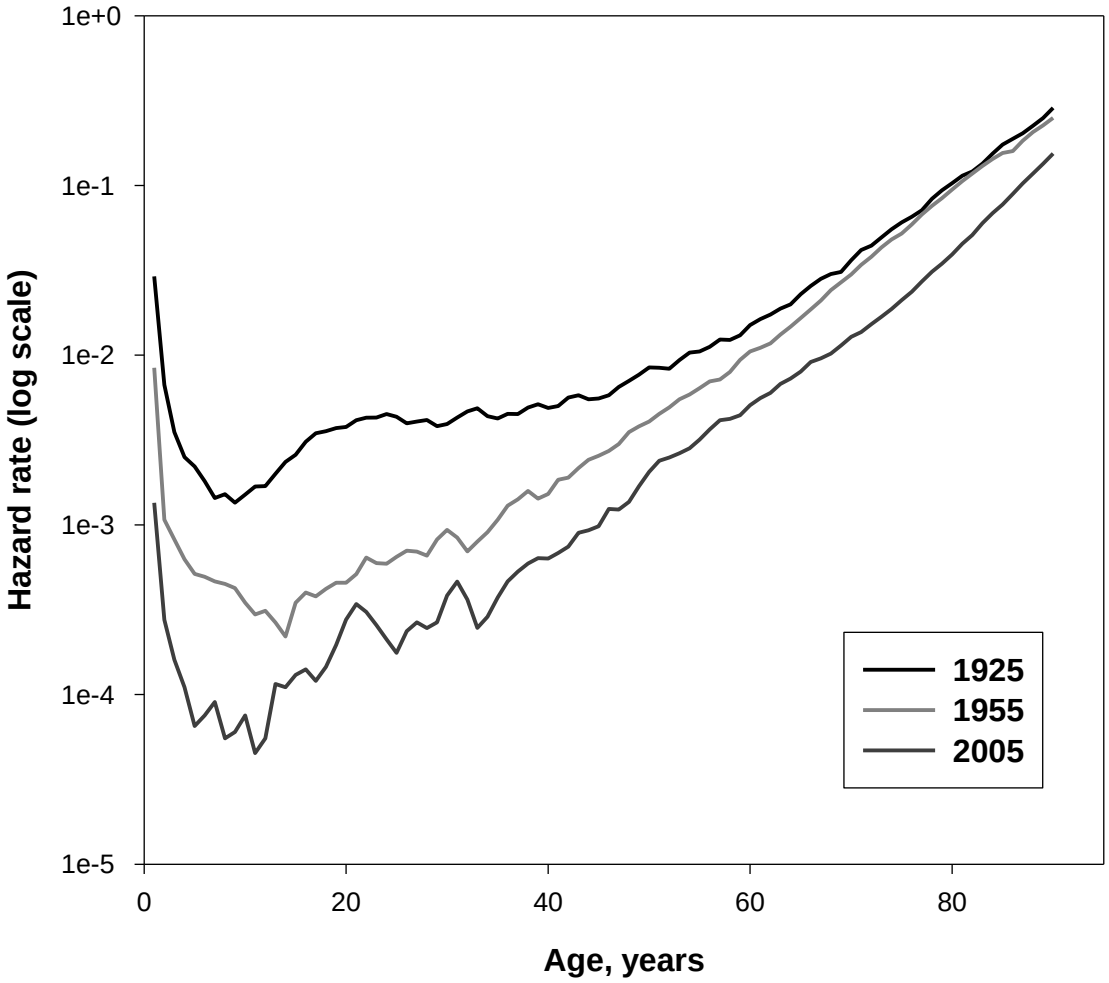

Figure 2. Changing patterns of mortality decline for Swedish females.

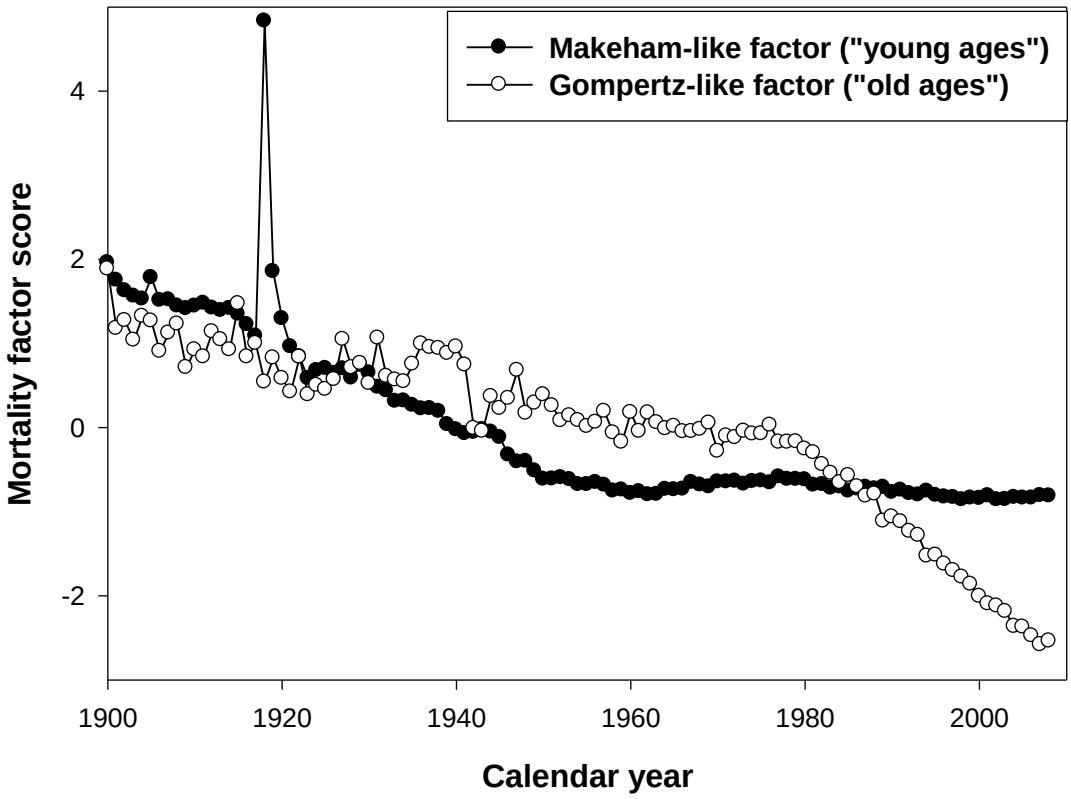

Figure 3. Time dependence of factor scores for “young-age” and “old-age” factors for Swedish males.

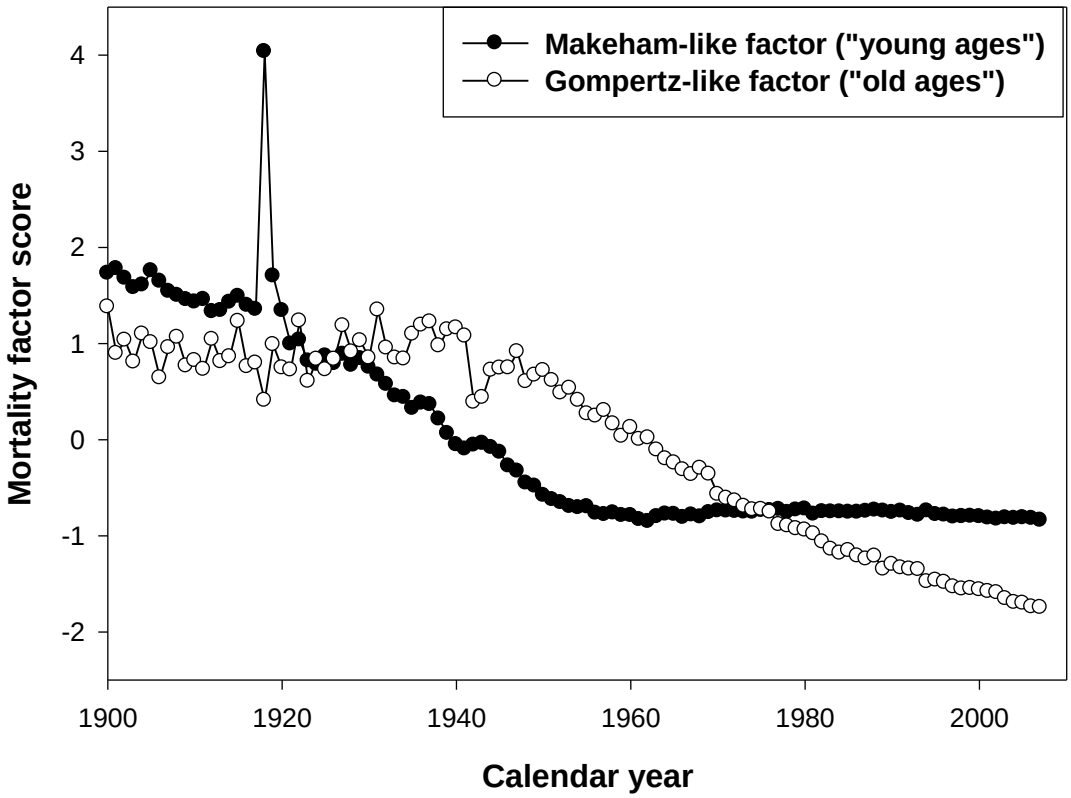

Figure 4. Time dependence of factor scores for “young-age” and “old-age” factors for Swedish females.

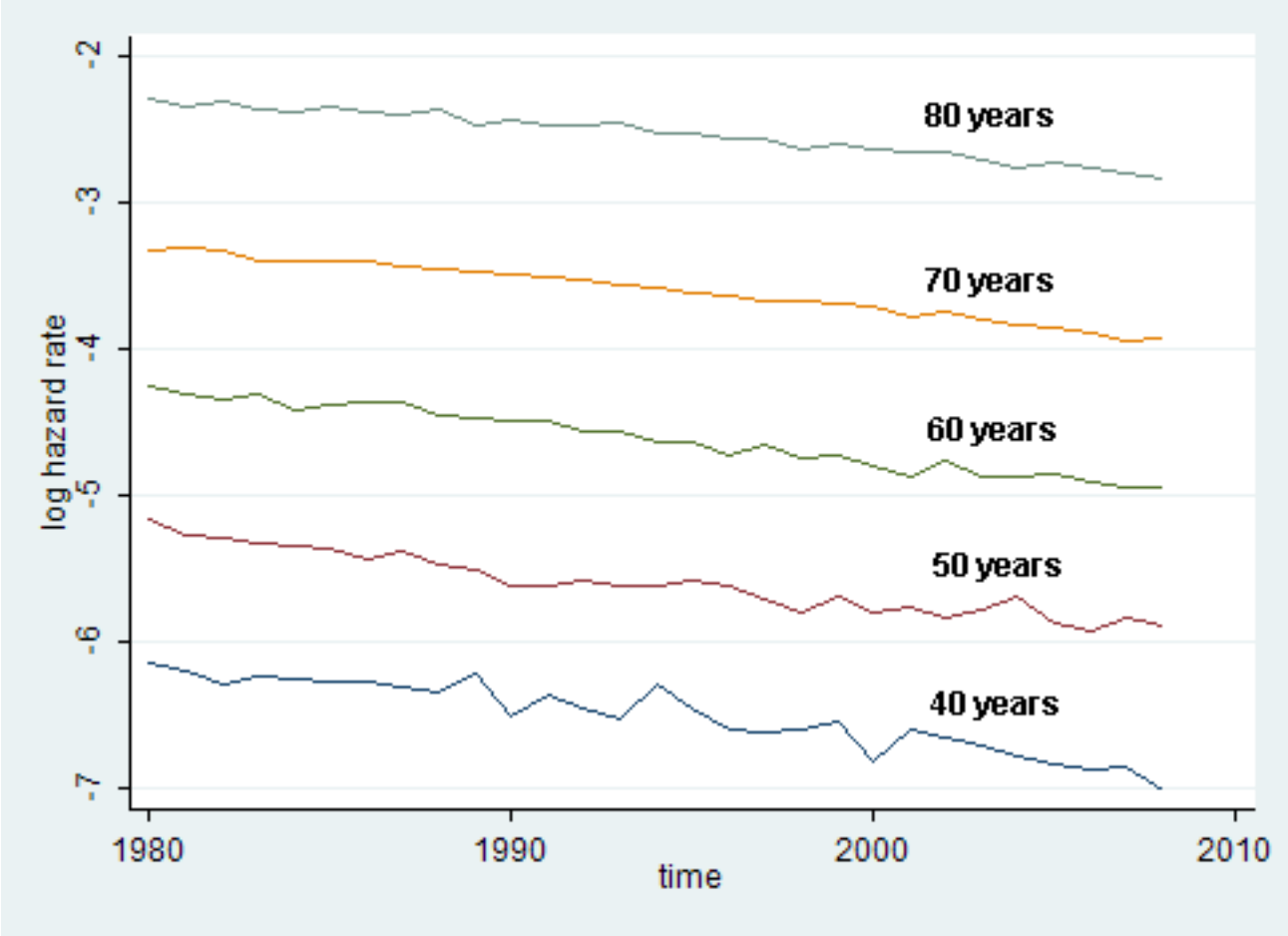

Figure 5. Linear changes of logarithm of hazard rate at different ages after 1980 for Swedish men.

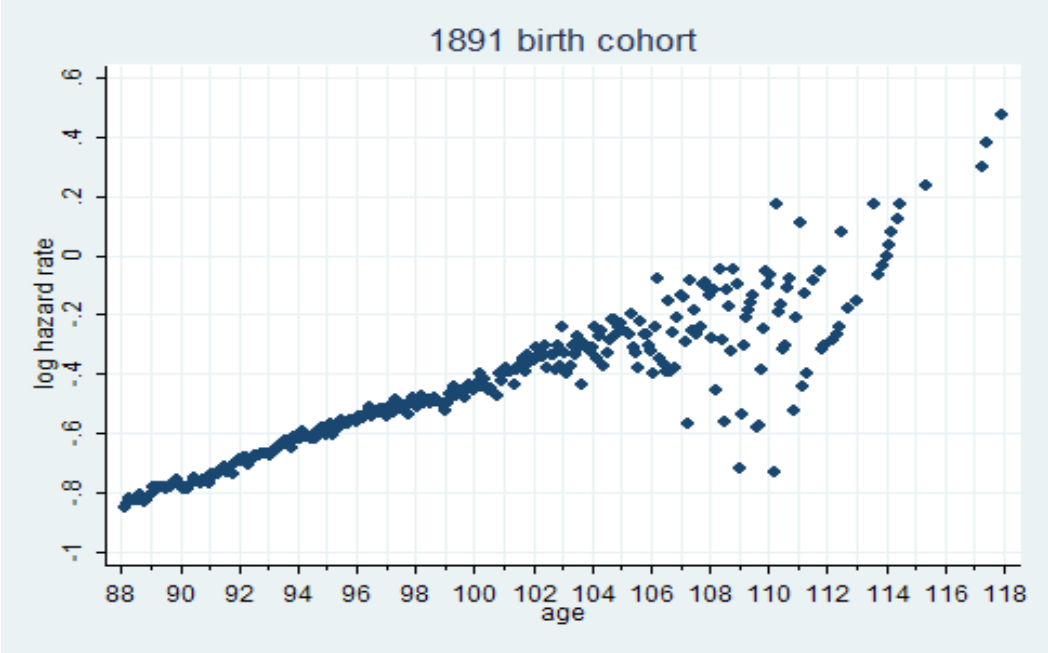

Figure 6. **Age-specific hazard rates (log scale) for U.S. population born in 1891. Data from the Social Security Administration Death Master File.**

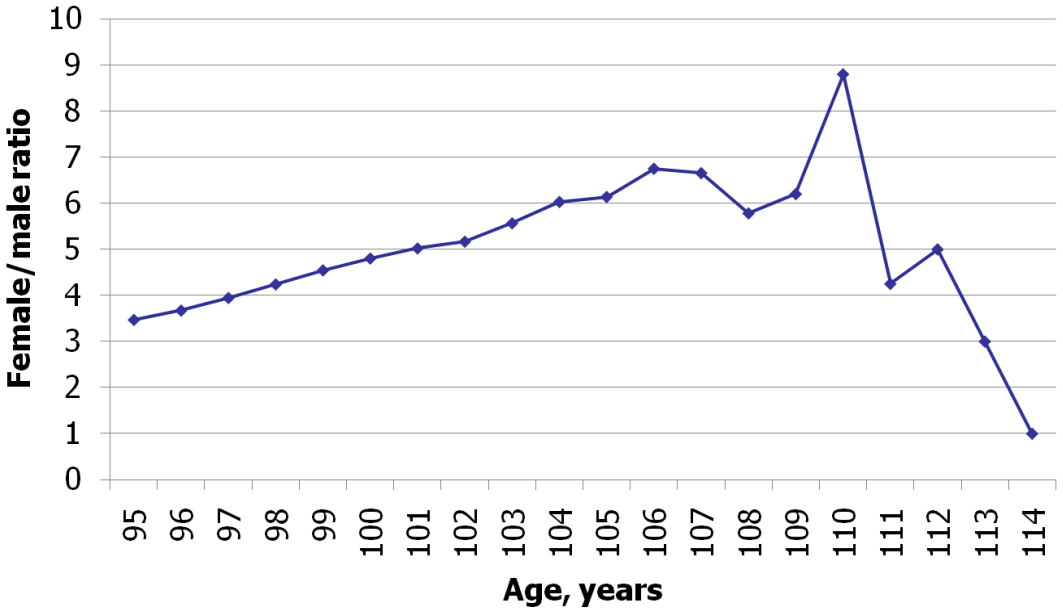

Figure 7. Observed female to male ratio at advanced ages for combined 1887-1892 birth cohort. If data are of good quality then this ratio should grow with age

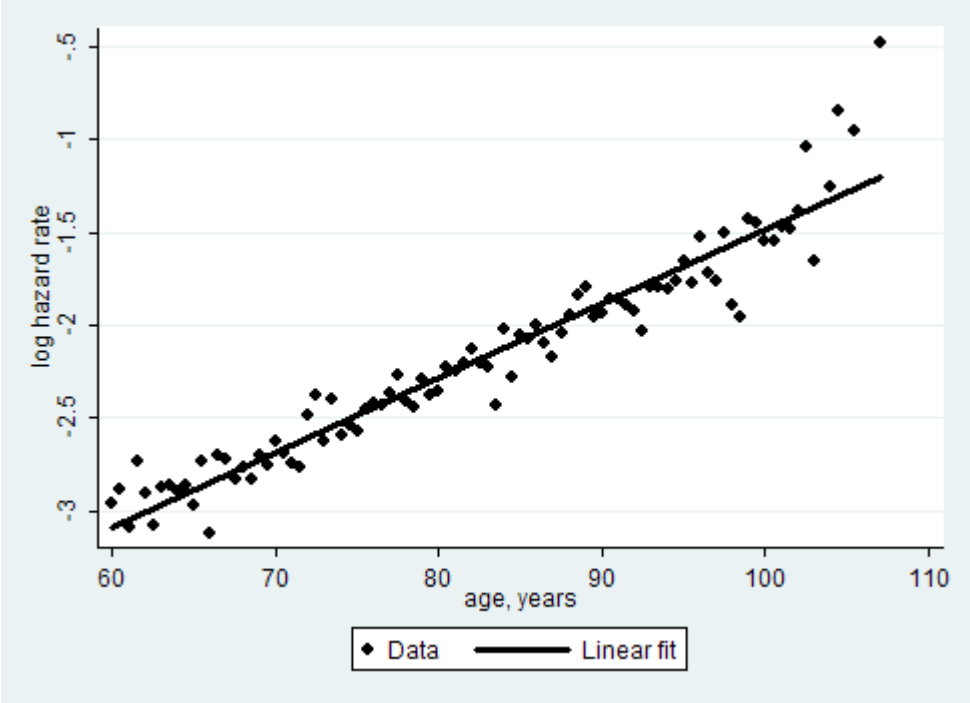

Figure 8. Age-specific hazard rate for 1681 siblings of centenarians born before 1880 and lived 60 years and more. Hazard rate was estimated for 6-month age intervals.

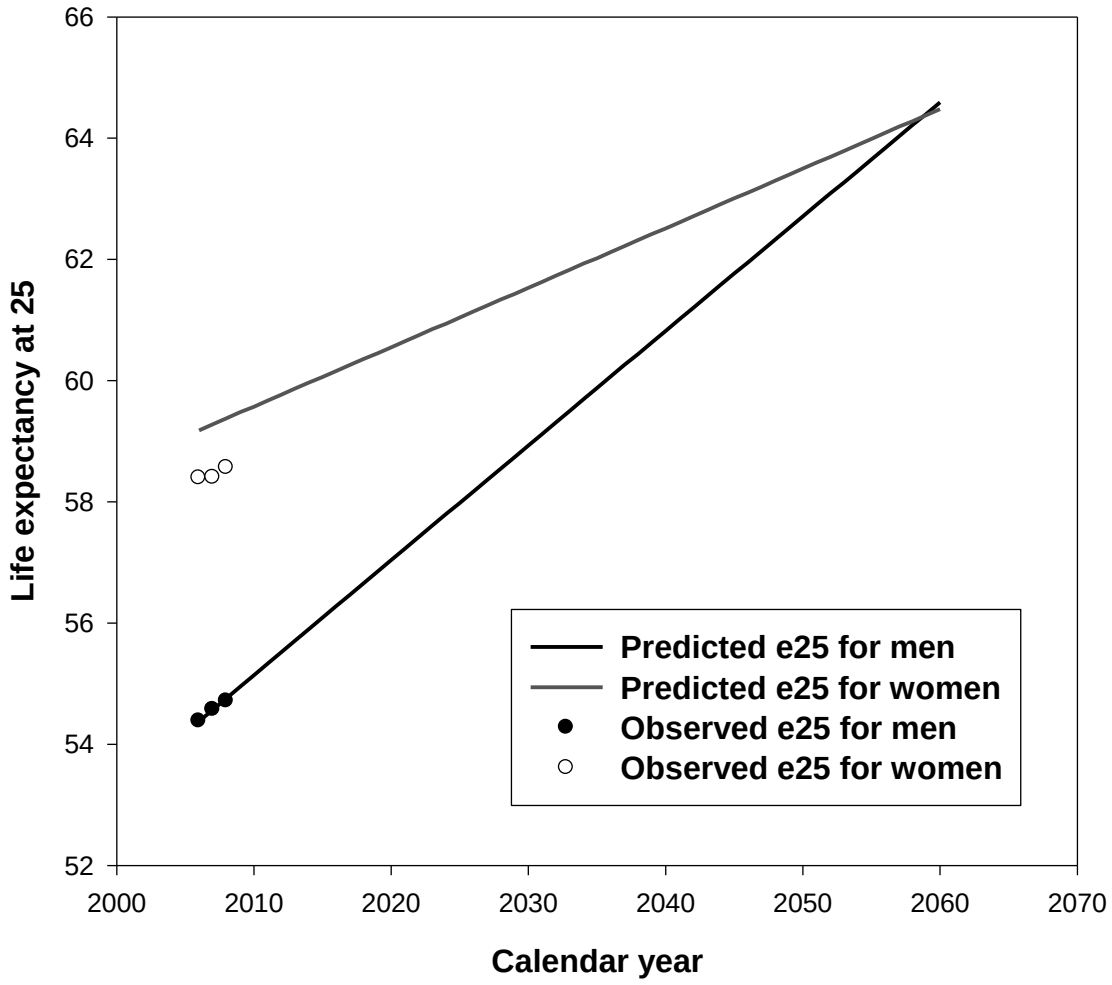

Figure 9. Projected trends of adult life expectancy (at 25 years) for Swedish men and women.

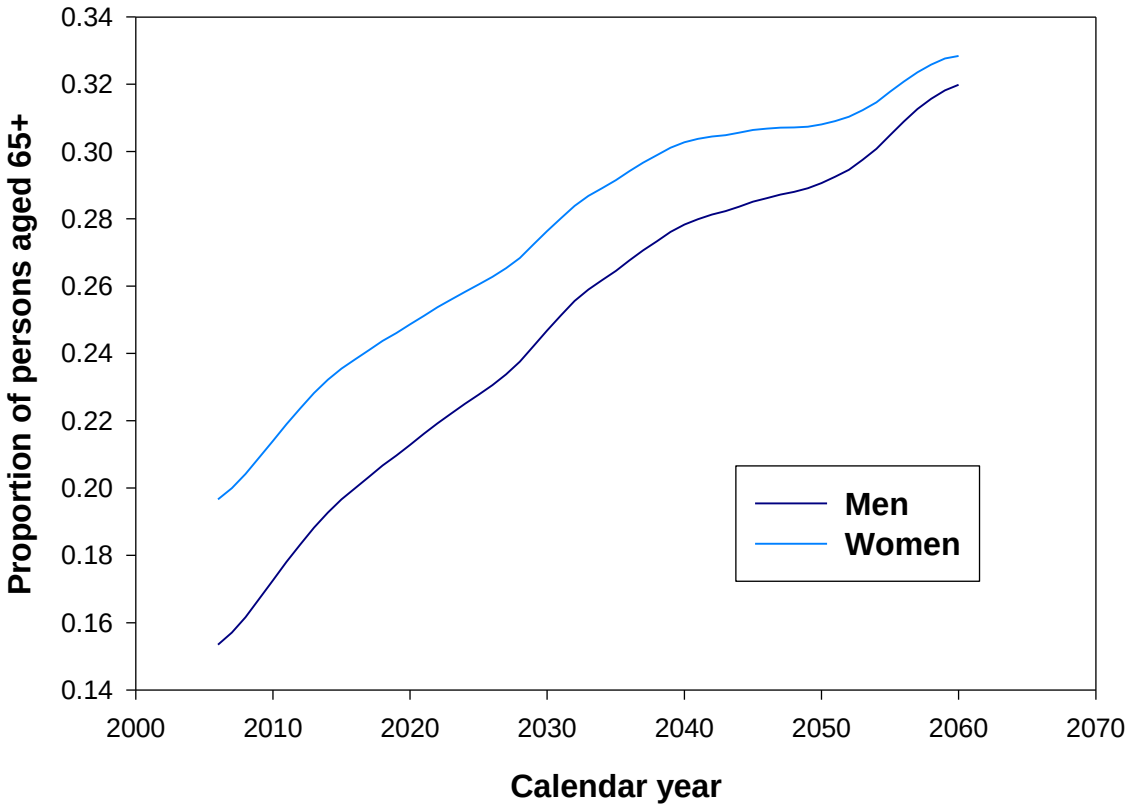

Figure 10. Projected changes in the proportion of older persons in Swedish population.

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