

# **Mortality Patterns at Advanced Ages**

Dr. Natalia S. Gavrilova, Ph.D.  
Dr. Leonid A. Gavrilov, Ph.D.

**Center on Aging  
NORC at The University of Chicago  
Chicago, Illinois, USA**

# Mortality at advanced ages is the key variable for understanding population trends among the oldest-old

THE WALL STREET JOURNAL

WSJ.com

THE NUMBERS GUY | March 2, 2012, 7:00 p.m. ET

## Death Gets in the Way of Old-Age Gains

By CARL BIALIK

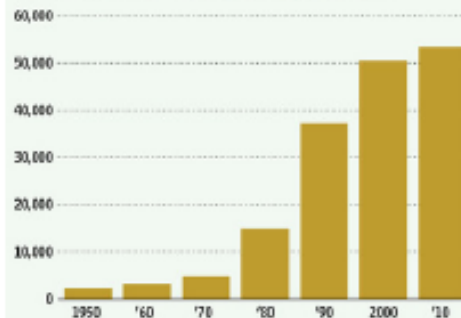


A new research paper, and a census surprise, are calling into question some long-held beliefs about a morbid bit of math: how much mortality rates increase with age.

It's no surprise that the older a group of people get, the higher the percentage of them who will die in any given time period. Benjamin Gompertz, a 19th-century British mathematician, charted the increase in mortality rates as very regular. His Gompertz law of mortality says that each additional period brings a constant percentage increase in mortality rates.

### Survivors

The increase in the number of centenarians in the U.S. has begun to slow, raising questions about gains in old-age survival.



Note: Numbers prior to 1998 are estimates, revised from census counts to fit later in data.  
Source: U.S. Census Bureau  
The Wall Street Journal

In the 20th century, though, as the world population aged and demographers' data improved, Gompertz started to look fallible. Researchers have found that, starting around age 80, mortality keeps increasing, but more slowly. More 100-year-olds die before turning 101 than 80-year-olds do before their 81st birthday, but the difference was less than Gompertz predicted.

But Gompertz may be right after all. In a study published last year and publicized last month, two longtime researchers of aging and believers in the late-life mortality slowdown reported that they and others were wrong. Death rates among Americans born between 1875 and 1895 kept on climbing steadily as they aged, they found, all the way through age 106, when their numbers got too sparse to follow.

This is bad news for anyone who wants to reach the century mark, but could provide an odd measure of relief for pensions, retirement programs and medical insurers, whose costs rise as people live longer.

**Recent projections of  
the U.S. Census Bureau  
significantly overestimated the  
actual number of centenarians**

# Views about the number of centenarians in the United States 2009

Centenarians are the fastest-growing age segment:  
Number of 100-year-olds to hit 6 million by 2050

BY THE ASSOCIATED PRESS

TUESDAY, JULY 21, 2009, 10:27 AM

# New estimates based on the 2010 census are two times lower than the U.S. Bureau of Census forecast

## Far fewer centenarians than expected in Census



Posted Sept. 24, 2011, at 6:19 a.m.

Last modified Sept. 24, 2011, at 7:06 a.m.

**NEW YORK** — Reports of Americans living beyond the ripe old age of 100, it appears, were greatly exaggerated.

The Census Bureau predicted six years ago that the country would be home to 114,000 centenarians by 2010. The actual number was 53,364, the census reported recently. That represented an increase of 5.8 percent since 2000, compared with a 9.7 percent gain in the nation's population as a whole.



# The same story recently happened in the Great Britain

**Financial Times**

September 11, 2012 8:20 pm

## Long-lived Britons increasing slower than forecast

By Norma Cohen, Economics Correspondent



The rate at which Britons are living into very old age is rising much more slowly than had been forecast only two years ago, a blow for those hoping for a very long life but good news for pension providers and the Treasury which spend hefty sums on the oldest old.

**Earlier studies suggested that the exponential growth of mortality with age (Gompertz law) is followed by a period of deceleration, with slower rates of mortality increase.**

# The Gompertz-Makeham Law

Death rate is a sum of age-independent component (Makeham term) and age-dependent component (Gompertz function), which increases exponentially with age.

$$\mu(x) = A + R e^{ax}$$

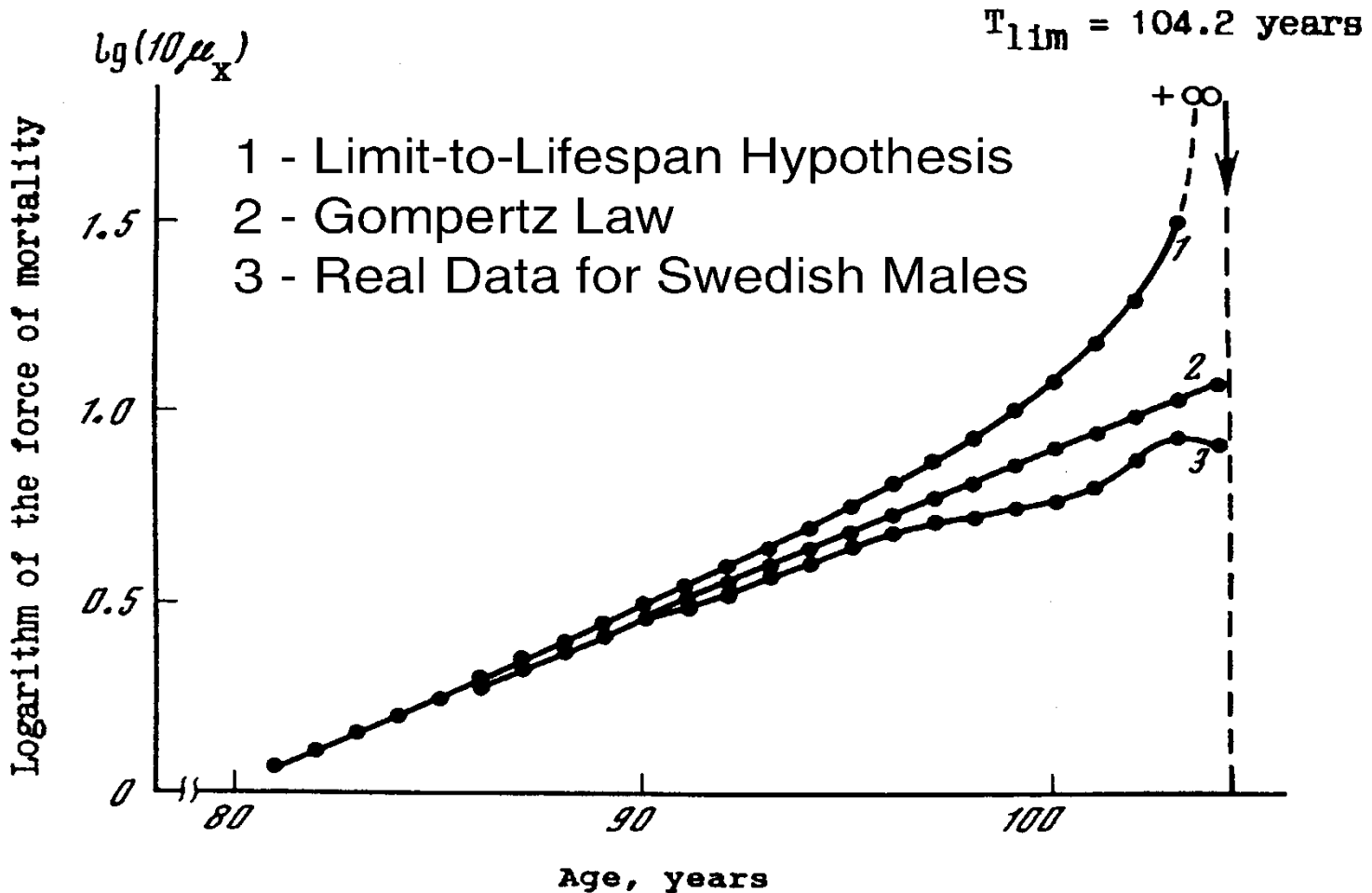
risk of death

$A$  – Makeham term or background mortality

$R e^{ax}$  – age-dependent mortality;  $x$  - age



# Mortality at Advanced Ages – over 20 years ago



Source: **Gavrilov L.A., Gavrilova N.S. The Biology of Life Span: A Quantitative Approach, NY: Harwood Academic Publisher, 1991**

**The first comprehensive  
study of mortality at  
advanced ages was  
published in 1939**

# **HUMAN BIOLOGY**

**a record of research**

**FEBRUARY, 1939**

**VOL. 11**



**No. 1**

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**THE BIostatISTICS OF SENILITY**

**BY MAJOR GREENWOOD AND J. O. IRWIN**

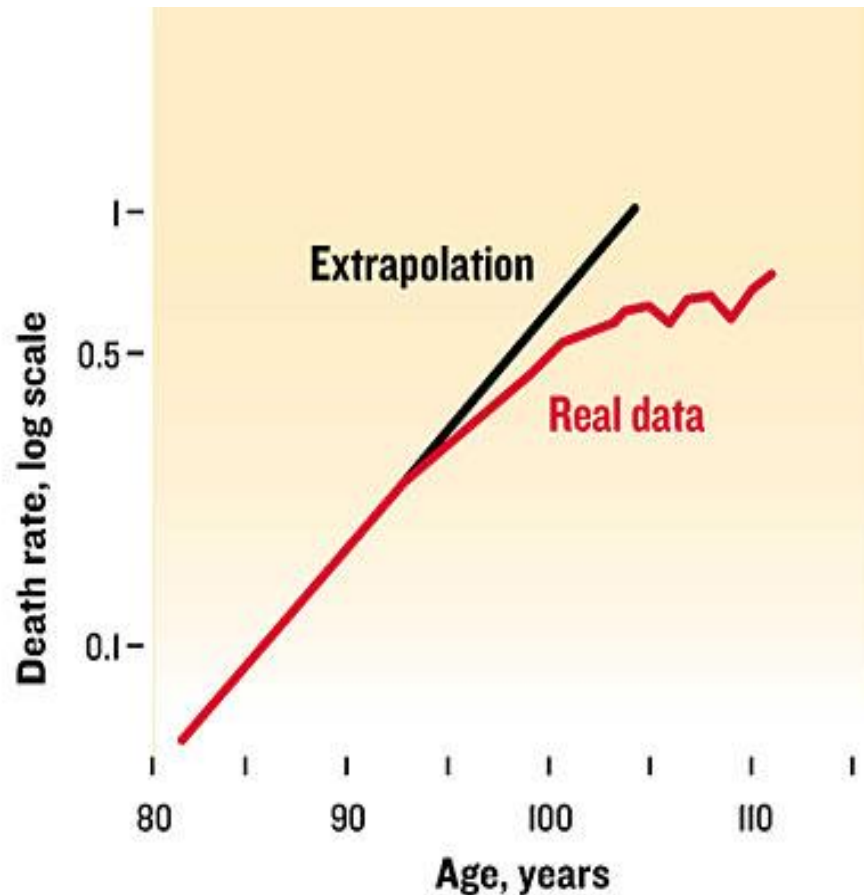
M. Greenwood, J. O. Irwin. BIOSTATISTICS OF SENILITY

" the increase of mortality rate with age advances at a slackening rate, that nearly all, perhaps all, methods of graduation of the type of Gompertz's formula *over-state* senile mortality. "

"... *possibility* that with advancing age the rate of mortality asymptotes to a finite value. "

"... The limiting values of  $q_{\infty}$  are 0.439 for women and 0.544 for men. Some tests of the ultimate mortalities in non-human experience were not unfavorable. "

# Mortality deceleration at advanced ages.

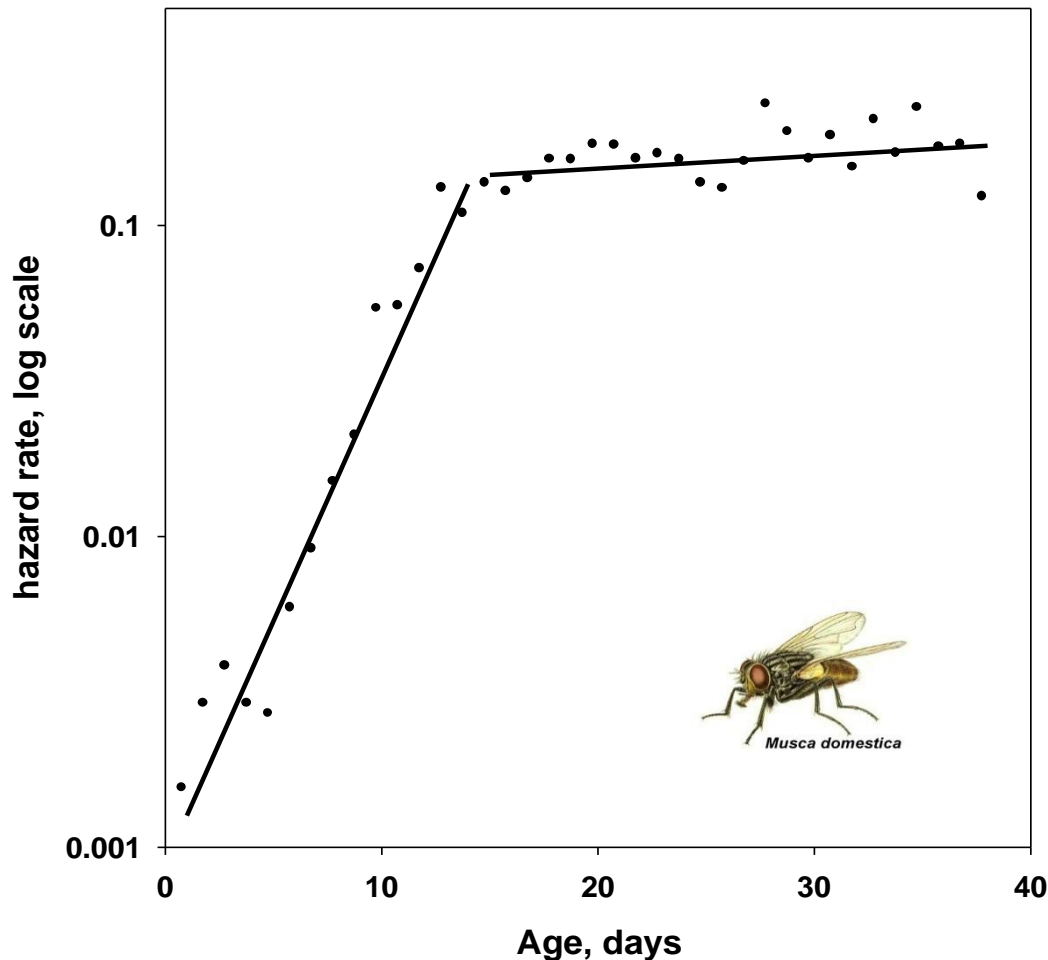


After age 95, the observed risk of death [red line] deviates from the values predicted by the Gompertz law [black line].

Mortality of Swedish women for the period of 1990-2000 from the Kannisto-Thatcher Database on Old Age Mortality

Source: Gavrilov, Gavrilova, "Why we fall apart. Engineering's reliability theory explains human aging". *IEEE Spectrum*. 2004.

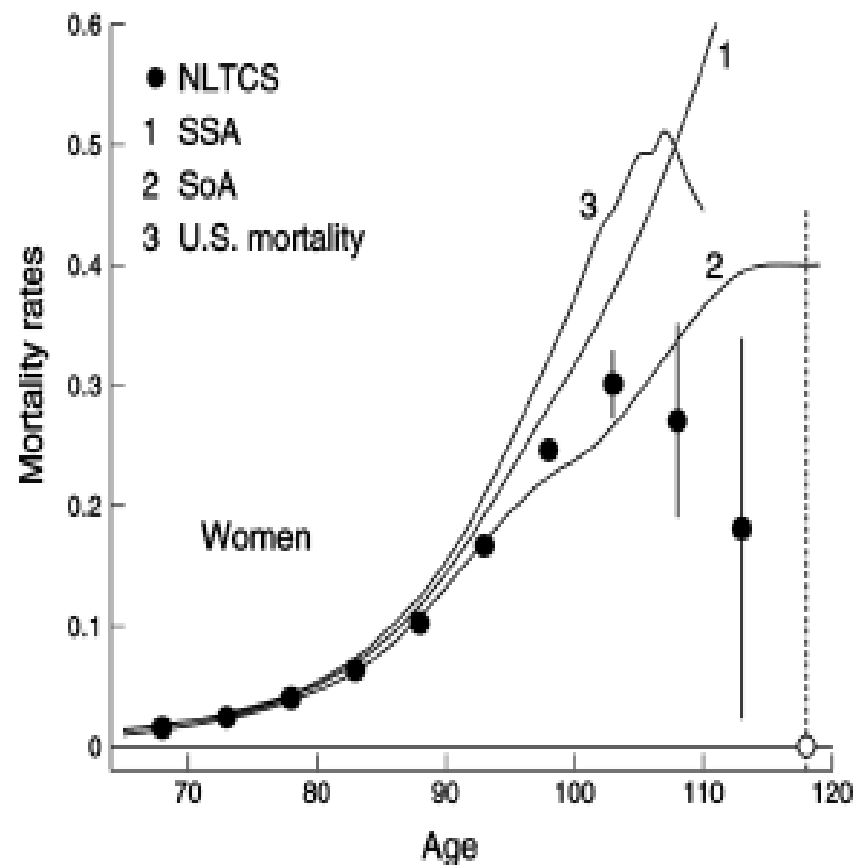
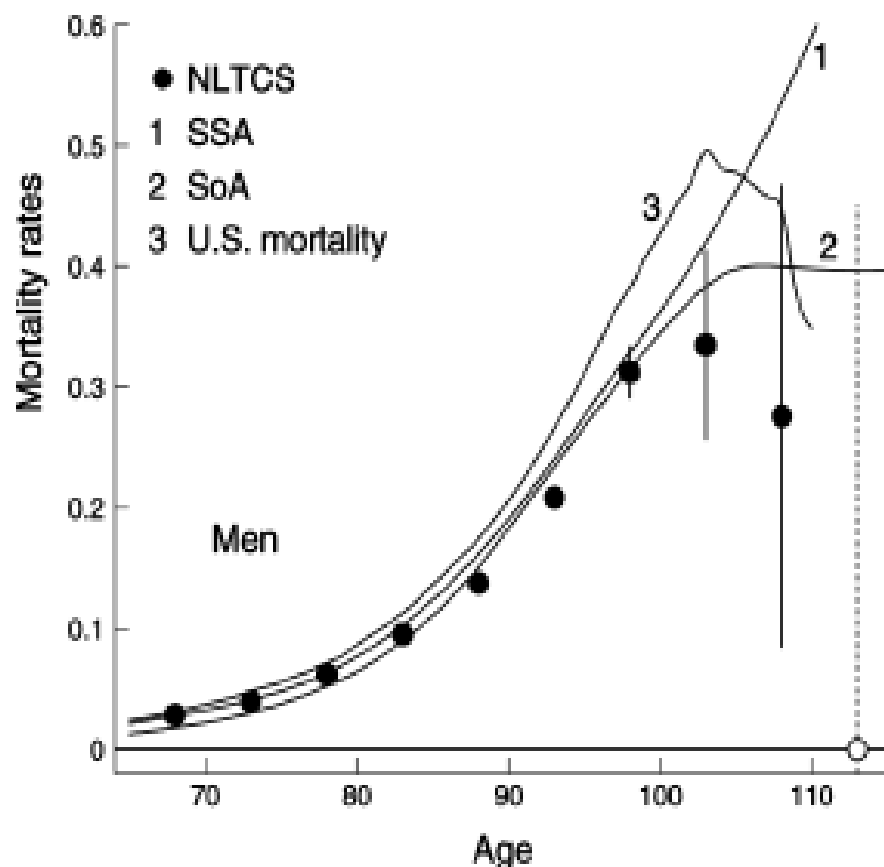
# Mortality Leveling-Off in House Fly *Musca domestica*



Based on life table of 4,650 male house flies published by Rockstein & Lieberman, 1959

Source: Gavrilov, Gavrilova, Handbook of the Biology of Aging, 2006

# Mortality at Advanced Ages, Recent Study



Source: Manton et al. (2008). Human Mortality at Extreme Ages: Data from the NLTCS and Linked Medicare Records. *Math.Pop.Studies*

# Existing Explanations of Mortality Deceleration

- Population Heterogeneity** (Beard, 1959; Sacher, 1966). "... sub-populations with the higher injury levels die out more rapidly, resulting in progressive selection for vigour in the surviving populations" (Sacher, 1966)
- Exhaustion of organism's redundancy** (reserves) at extremely old ages so that every random hit results in death (Gavrilov, Gavrilova, 1991; 2001)
- Lower risks of death for older people** due to less risky behavior (Greenwood, Irwin, 1939)
- Evolutionary explanations** (Mueller, Rose, 1996; Charlesworth, 2001)



# **Study of the Social Security Administration Death Master File**

## **MORTALITY MEASUREMENT AT ADVANCED AGES: A STUDY OF THE SOCIAL SECURITY ADMINISTRATION DEATH MASTER FILE**

Leonid A. Gavrilov\* and Natalia S. Gavrilova†

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### **ABSTRACT**

Accurate estimates of mortality at advanced ages are essential to improving forecasts of mortality and the population size of the oldest old age group. However, estimation of hazard rates at extremely old ages poses serious challenges to researchers: (1) The observed mortality deceleration

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**NORTH AMERICAN ACTUARIAL JOURNAL, VOLUME 15, NUMBER 3**

***North American Actuarial Journal, 2011,  
15(3):432-447***

# **What Is SSA's DMF ?**

**As a result of a court case under the Freedom of Information Act, SSA is required to release its death information to the public. SSA's DMF contains the complete and official SSA database extract, as well as updates to the full file of persons reported to SSA as being deceased.**

**SSA DMF is no longer a publicly available data resource (now is available from Ancestry.com for fee)**

**We used DMF full file obtained from the National Technical Information Service (NTIS). Last deaths occurred in September 2011.**

# **SSA's DMF Advantage**

**Some birth cohorts covered by DMF could be studied by the method of extinct generations**

**Considered superior in data quality compared to vital statistics records by some researchers**

**Mortality force (hazard rate) is the best indicator to study mortality at advanced ages**

$$\mu_x = -\frac{dN_x}{N_x dx} = -\frac{d \ln(N_x)}{dx} \approx -\frac{\Delta \ln(N_x)}{\Delta x}$$

**Does not depend on the length of age interval**

**Has no upper boundary and theoretically can grow unlimitedly**

**Famous Gompertz law was proposed for fitting age-specific mortality force function (Gompertz, 1825)**

# **Problems in Hazard Rate Estimation At Extremely Old Ages**

- 1. Mortality deceleration in humans may be an artifact of mixing different birth cohorts with different mortality (heterogeneity effect)**
- 2. Standard assumptions of hazard rate estimates may be invalid when risk of death is extremely high**
- 3. Ages of very old people may be highly exaggerated**

# **Social Security Administration's Death Master File (SSA's DMF) Helps to Alleviate the First Two Problems**

**Allows to study mortality in large, more homogeneous single-year or even single-month birth cohorts**

**Allows to estimate mortality in one-month age intervals narrowing the interval of hazard rates estimation**

# **Social Security Administration's Death Master File (DMF) Was Used in This Study:**

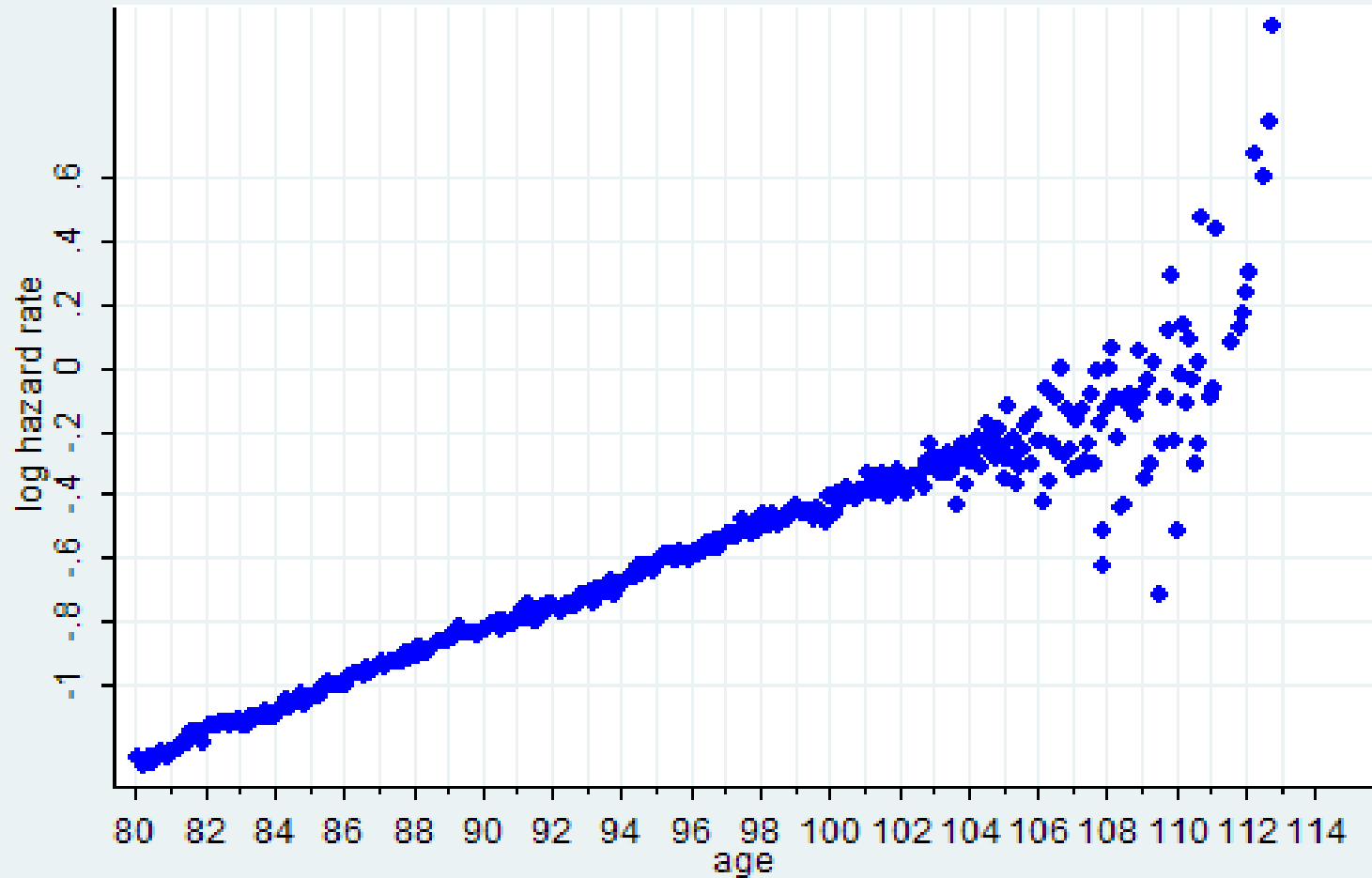
**To estimate hazard rates for relatively homogeneous single-year extinct birth cohorts (1890-1899)**

**To obtain monthly rather than traditional annual estimates of hazard rates**

**To identify the age interval and cohort with reasonably good data quality and compare mortality models**

# More recent birth cohort mortality

1898 birth cohort, females



Nelson-Aalen monthly estimates of hazard rates using Stata 11



# Hypothesis

**Mortality deceleration at advanced ages among DMF cohorts may be caused by poor data quality (age exaggeration) at very advanced ages**

**If this hypothesis is correct then mortality deceleration at advanced ages should be less expressed for data with better quality**

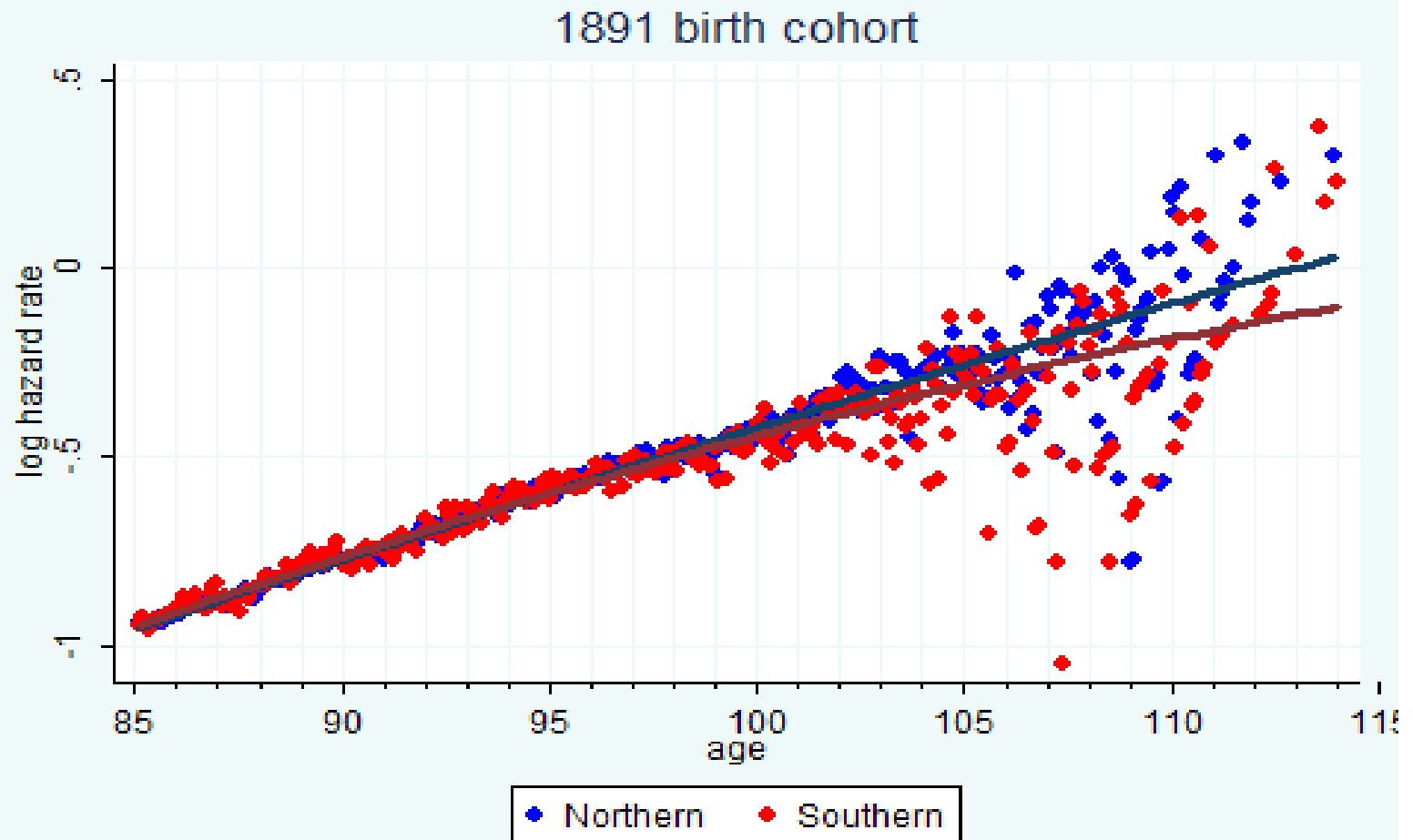
# **Quality Control (1)**

**Study of mortality in the states with different quality of age reporting:**

**Records for persons applied to SSN in the Southern states were found to be of lower quality (Rosenwaike, Stone, 2003)**

**We compared mortality of persons applied to SSN in Southern states, Hawaii, Puerto Rico, CA and NY with mortality of persons applied in the Northern states (the remainder)**

# Mortality for data with presumably different quality: Southern and Non-Southern states of SSN receipt



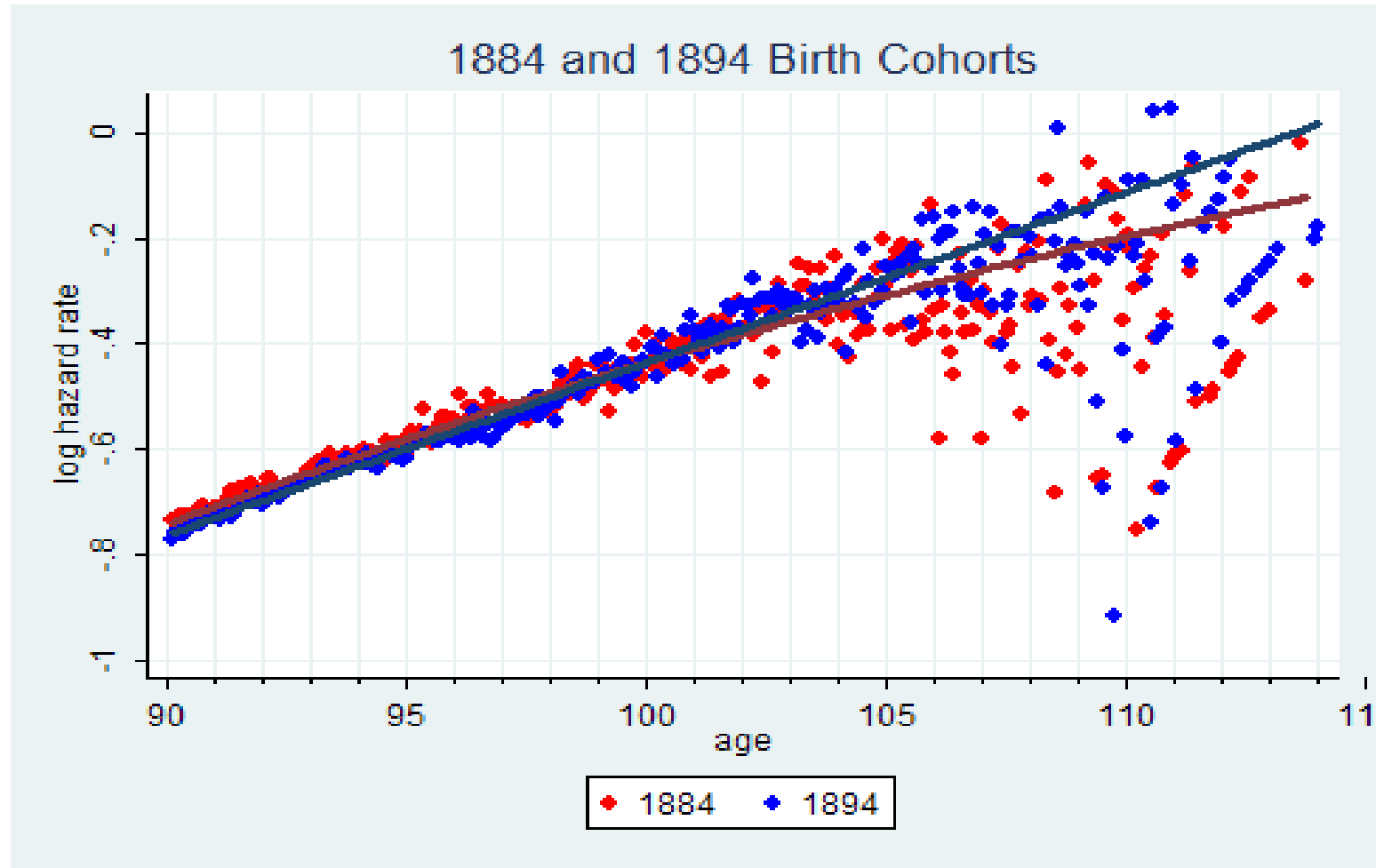
The degree of deceleration was evaluated using quadratic model

# Quality Control (2)

**Study of mortality for earlier and later single-year extinct birth cohorts:**

**Records for later born persons are supposed to be of better quality due to improvement of age reporting over time.**

# Mortality for data with presumably different quality: Older and younger birth cohorts

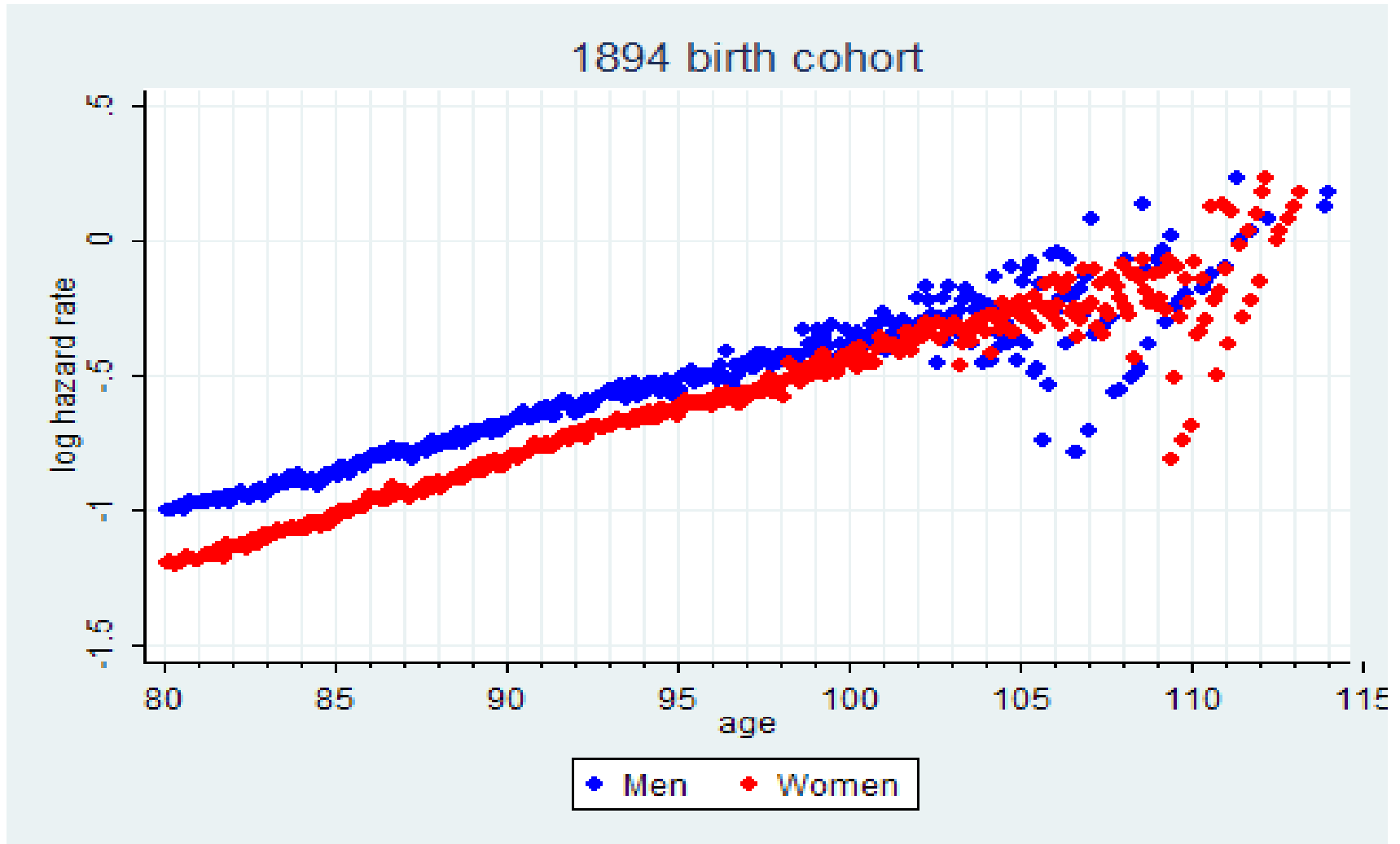


The degree of deceleration was evaluated using quadratic model

# **At what age interval data have reasonably good quality?**

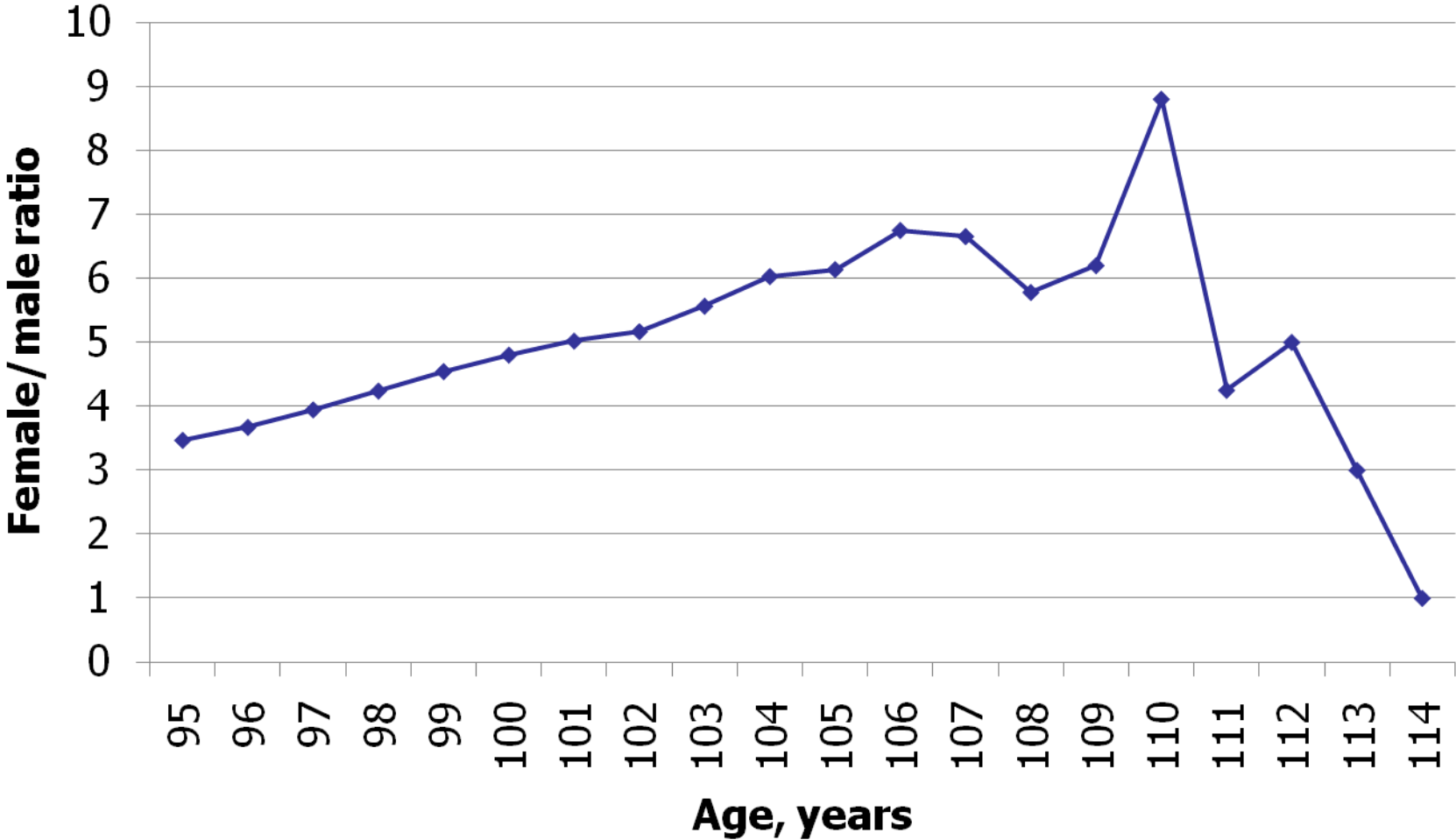
**A study of age-specific mortality by gender**

# Women have lower mortality at advanced ages



Hence number of females to number of males ratio should grow with age

# Observed female to male ratio at advanced ages for combined 1887-1892 birth cohort





# **Selection of competing mortality models using DMF data**

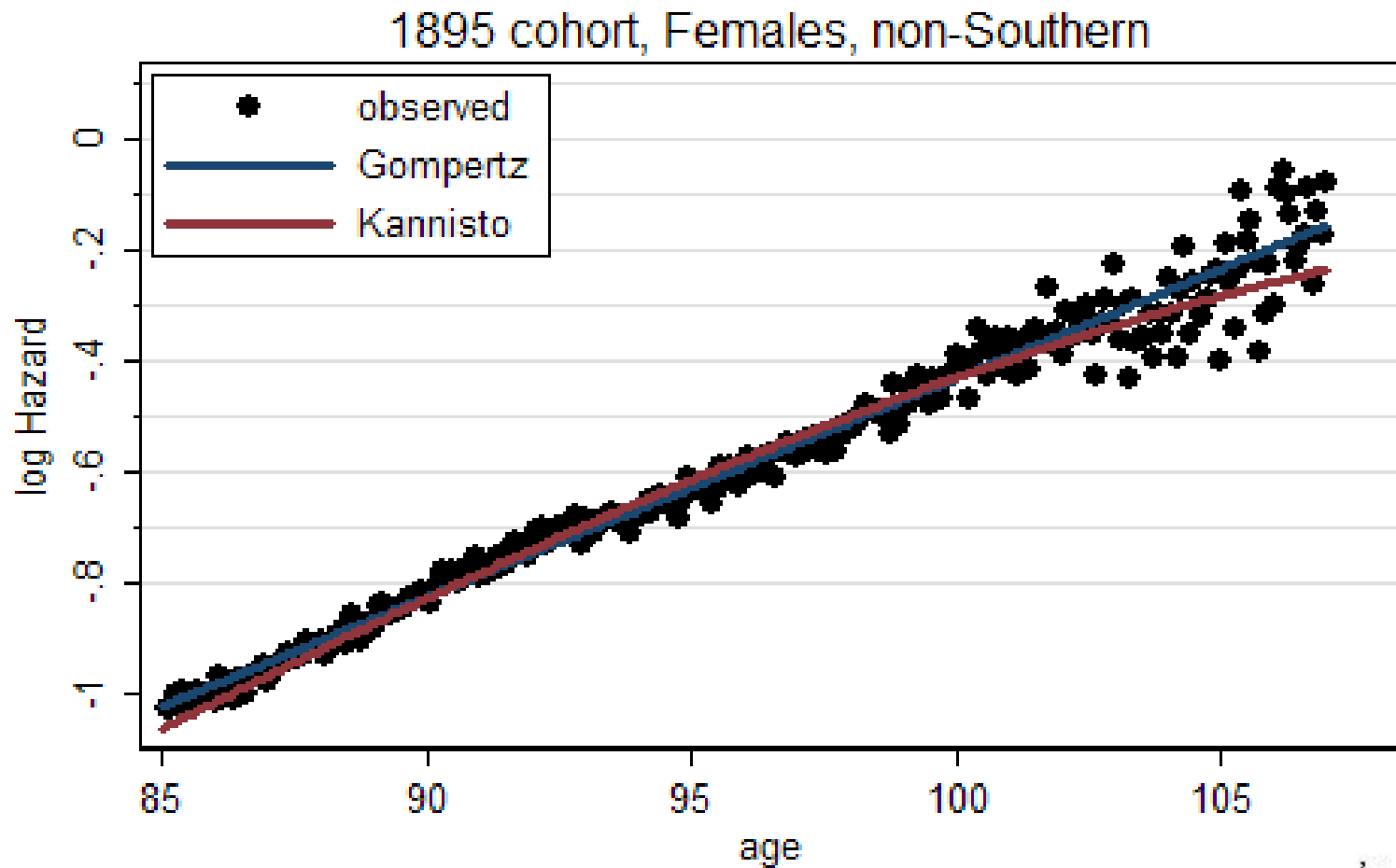
**Data with reasonably good quality were used: non-Southern states and 85-106 years age interval**

**Gompertz and logistic (Kannisto) models were compared**

**Nonlinear regression model for parameter estimates (Stata 11)**

**Model goodness-of-fit was estimated using AIC and BIC (Akaike and Bayesian Information Criteria)**

# Fitting mortality with Kannisto and Gompertz models



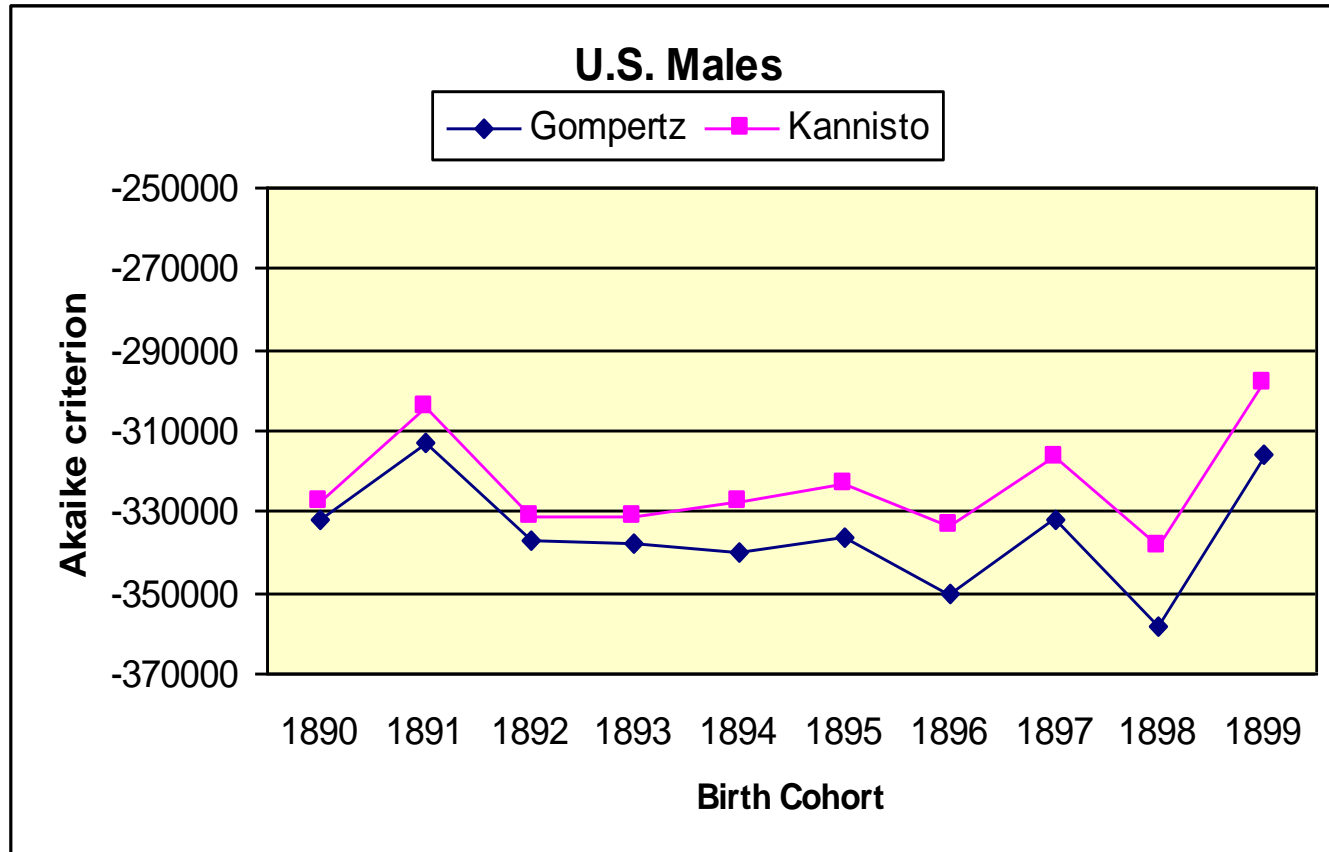
Gompertz  
model

$$\mu_x = ae^{bx}$$

Kannisto  
model

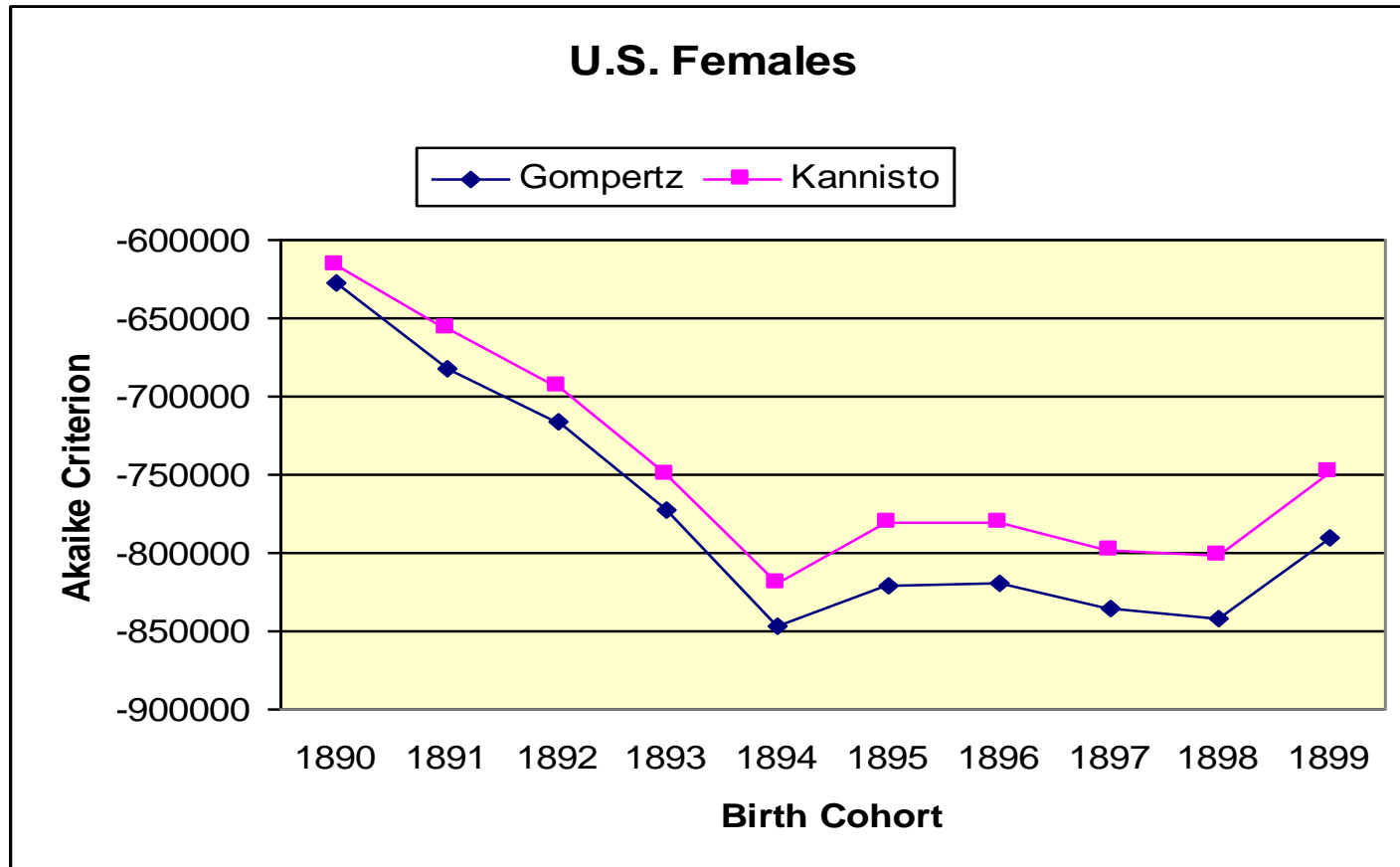
$$\mu_x = \frac{ae^{bx}}{1 + ae^{bx}}$$

# Akaike information criterion (AIC) to compare Kannisto and Gompertz models, men, by birth cohort (non-Southern states)



**Conclusion: In all ten cases Gompertz model demonstrates better fit than logistic model for men in age interval 85-106 years**

# Akaike information criterion (AIC) to compare Kannisto and Gompertz models, women, by birth cohort (non-Southern states)



**Conclusion: In all ten cases Gompertz model demonstrates better fit than logistic model for men in age interval 85-106 years**

# **Conclusions from our study of Social Security Administration Death Master File**

**Mortality deceleration at advanced ages among DMF cohorts is more expressed for data of lower quality**

**Mortality data beyond ages 106-107 years have unacceptably poor quality (as shown using female-to-male ratio test). The study by other authors also showed that beyond age 110 years the age of individuals in DMF cohorts can be validated for less than 30% cases (Young et al., 2010)**

**Source: Gavrilov, Gavrilova, *North American Actuarial Journal*, 2011, 15(3):432-447**

# The second studied dataset: U.S. cohort death rates taken from the Human Mortality Database

*Journals of Gerontology: BIOLOGICAL SCIENCES*  
Cite journal as: *J Gerontol A Biol Sci Med Sci*  
doi:10.1093/gerona/глу009

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## Biodemography of Old-Age Mortality in Humans and Rodents

Natalia S. Gavrilova and Leonid A. Gavrilov

Center on Aging, NORC at the University of Chicago, Chicago, Illinois.

Address correspondence to Natalia S. Gavrilova, PhD, Center on Aging, NORC at the University of Chicago, 1155 East 60th Street, Chicago, IL 60637.  
Email: [gavrilova@longevity-science.org](mailto:gavrilova@longevity-science.org)

The growing number of persons living beyond age 80 underscores the need for accurate measurement of mortality at advanced ages and understanding the old-age mortality trajectories. It is believed that exponential growth of mortality

**The second studied dataset:  
U.S. cohort death rates taken from  
the Human Mortality Database**

# **Selection of competing mortality models using HMD data**

**Data with reasonably good quality were used:  
80-106 years age interval**

**Gompertz and logistic (Kannisto) models were compared**

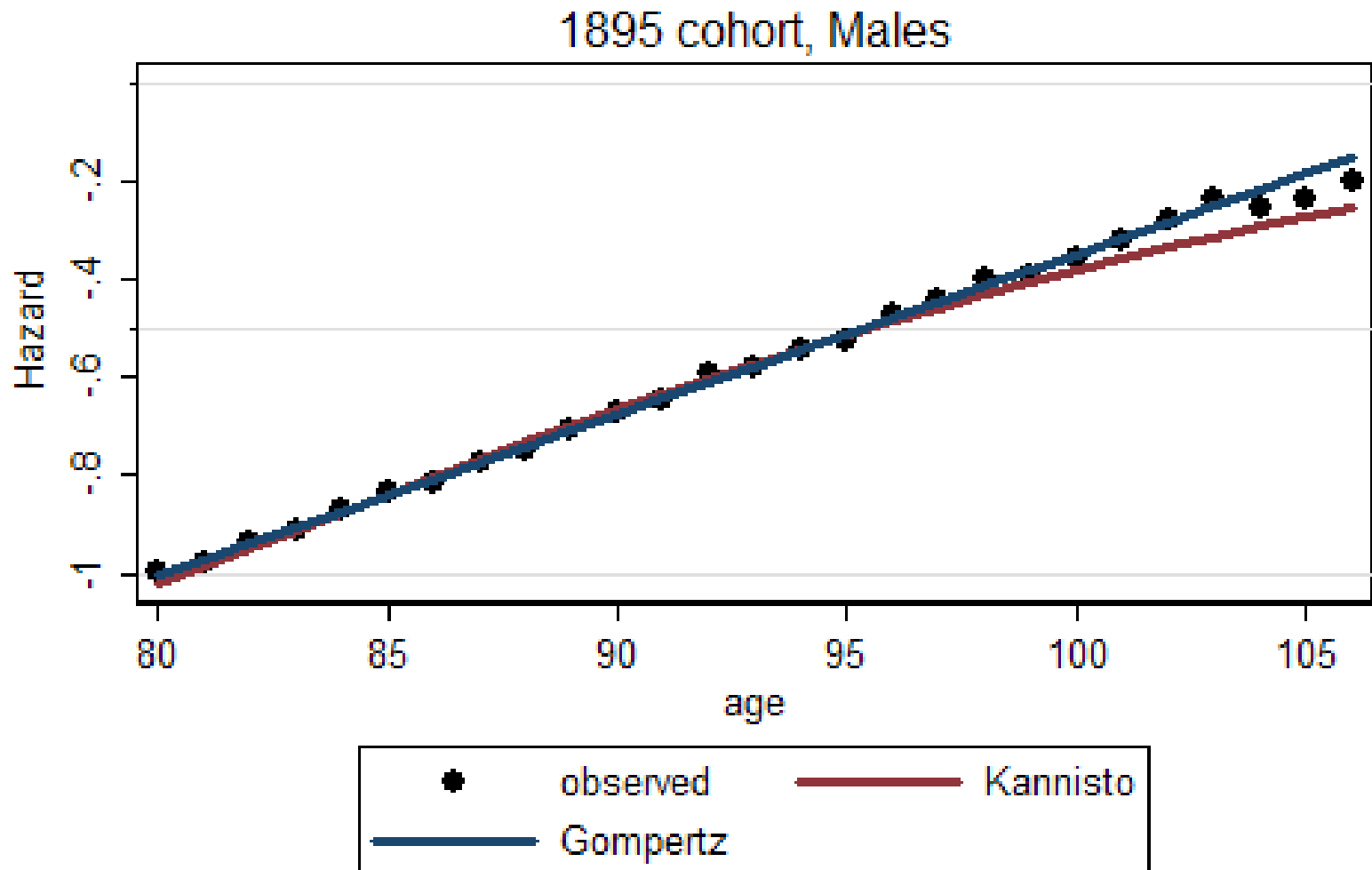
**Nonlinear weighted regression model for parameter estimates (Stata 11)**

**Age-specific exposure values were used as weights (Muller et al., Biometrika, 1997)**

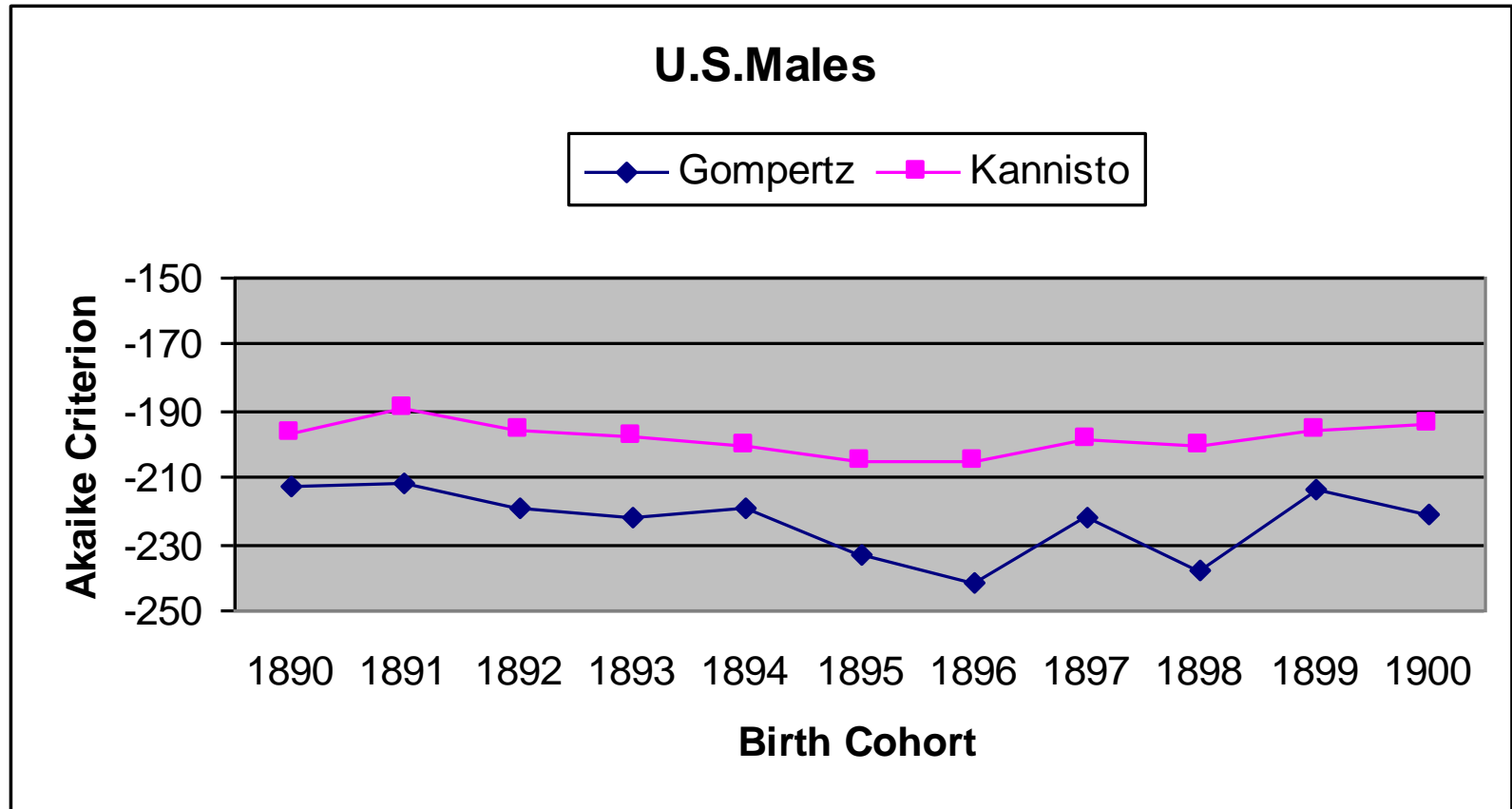
**Model goodness-of-fit was estimated using AIC and BIC**



# Fitting mortality with Kannisto and Gompertz models, HMD U.S. data

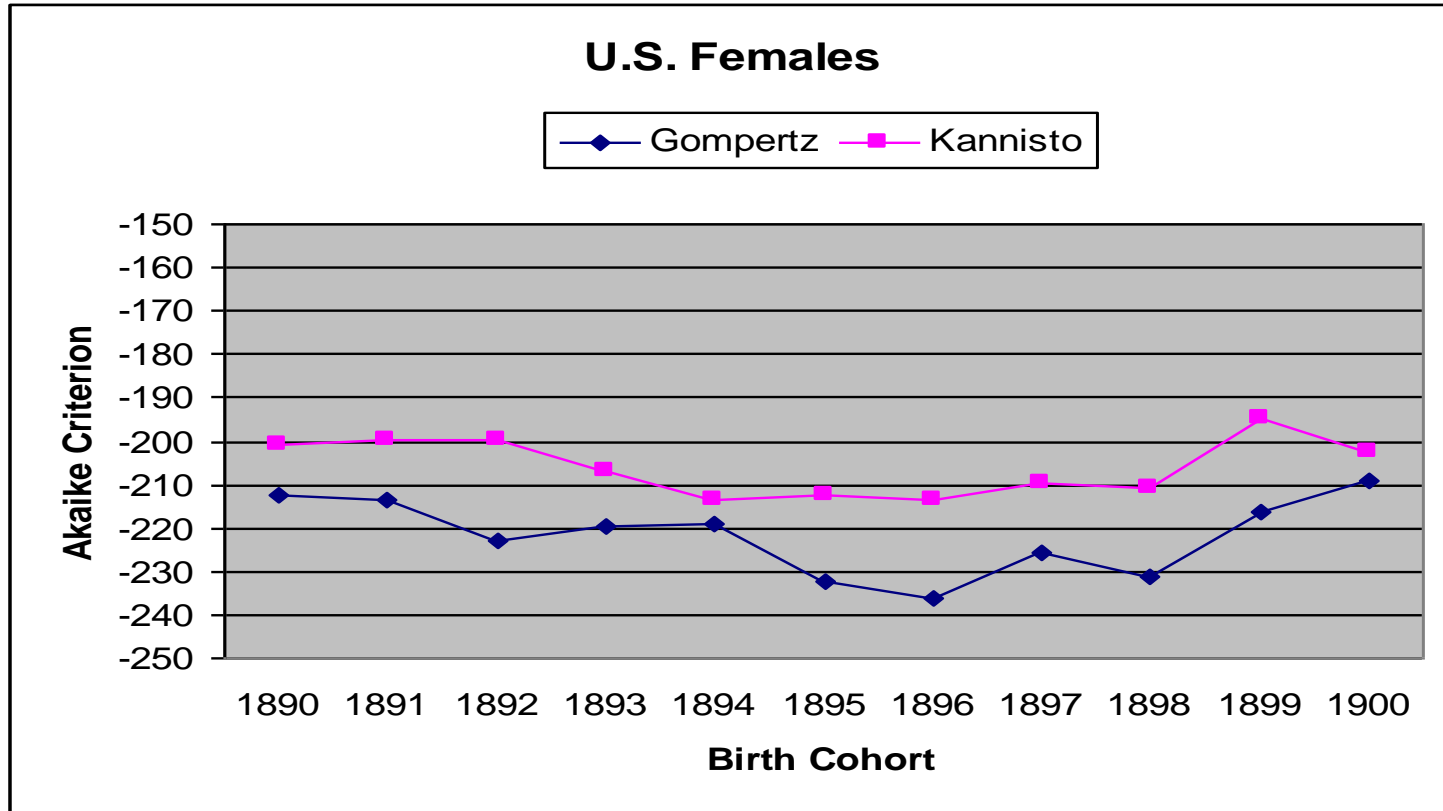


# Akaike information criterion (AIC) to compare Kannisto and Gompertz models, men, by birth cohort (HMD U.S. data)



**Conclusion: In all ten cases Gompertz model demonstrates better fit than logistic model for men in age interval 80-106 years**

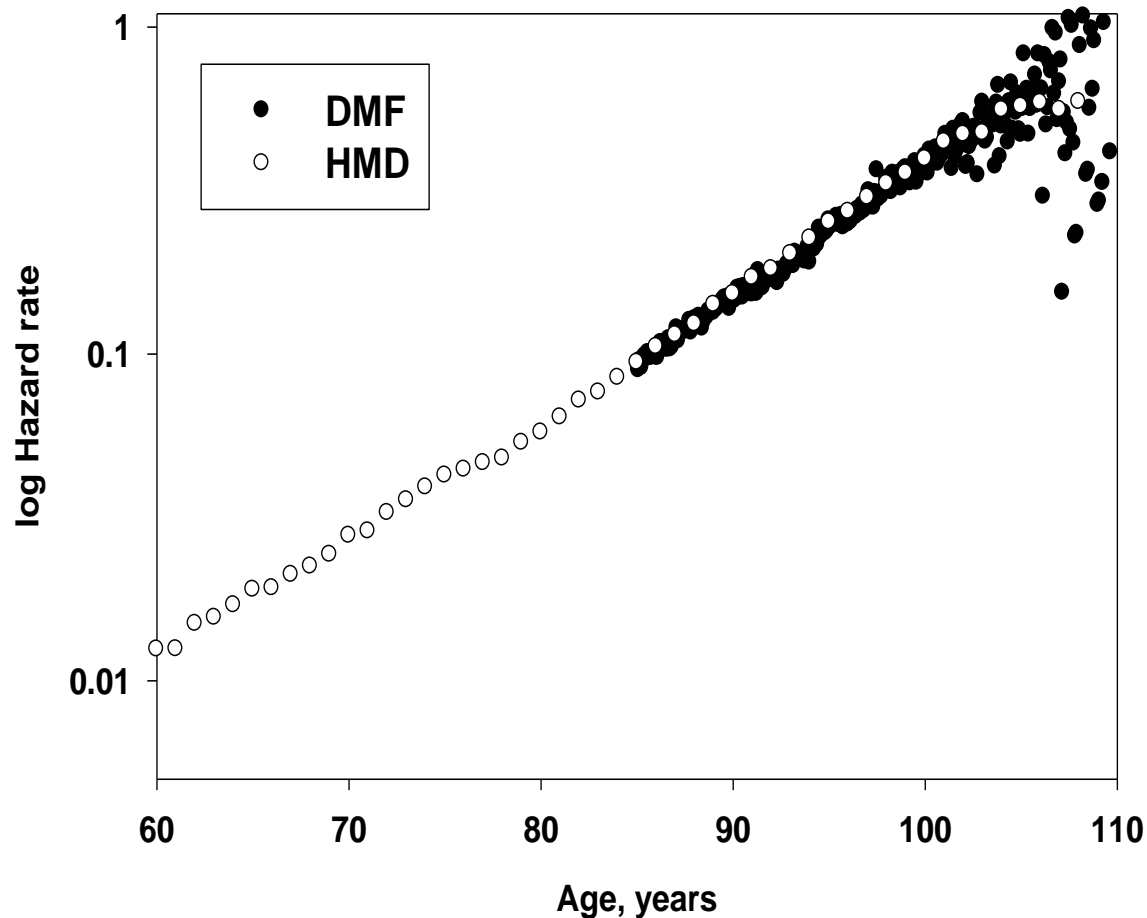
# Akaike information criterion (AIC) to compare Kannisto and Gompertz models, women, by birth cohort (HMD U.S. data)



**Conclusion: In all ten cases Gompertz model demonstrates better fit than logistic model for men in age interval 80-106 years**

# Compare DMF and HMD data

## Females, 1898 birth cohort



Hypothesis about two-stage Gompertz model is not supported by real data

# **Is Mortality Deceleration Caused by Age Misreporting?**

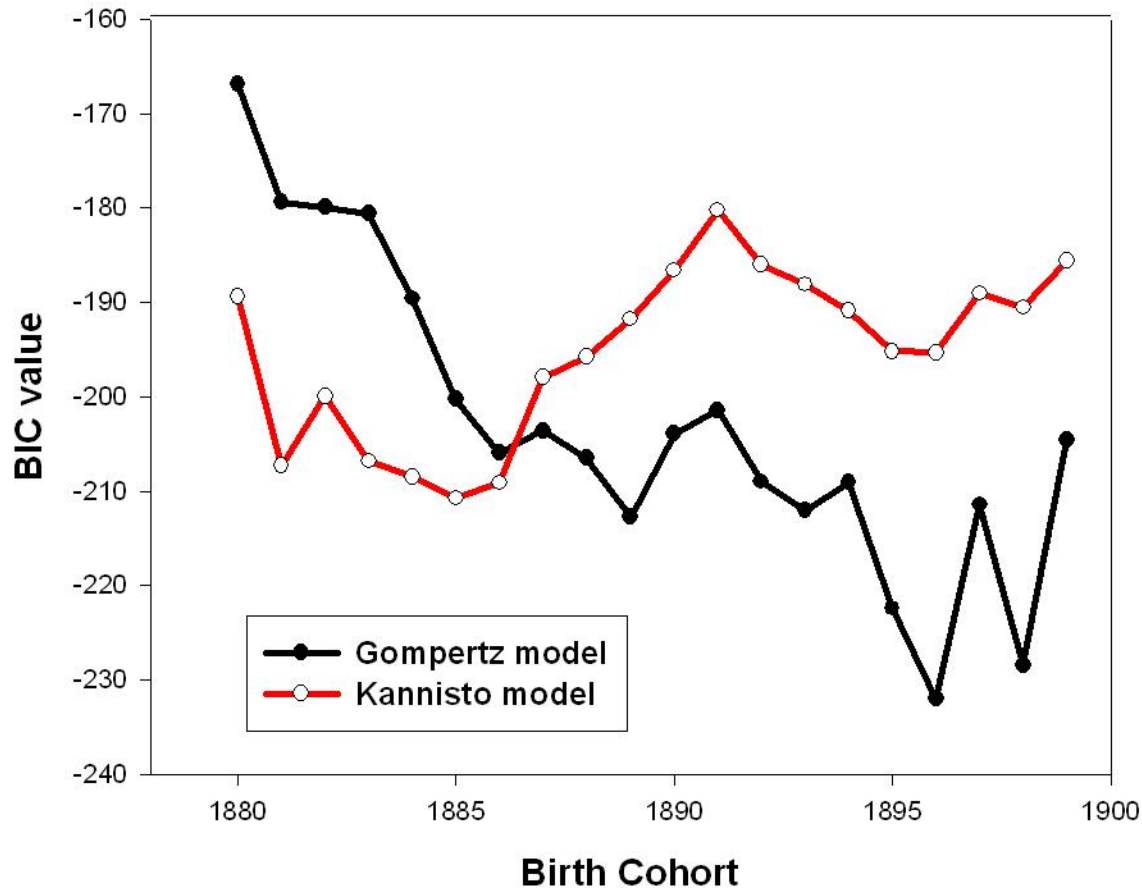
**It was demonstrated that age misstatement biases mortality estimates downwards at the oldest ages, which contributes to mortality deceleration (Preston et al., 1999).**

**If this hypothesis is correct then mortality deceleration should be more prevalent among historically older birth cohorts**

# Historical Evolution of Mortality Trajectories

1880-1899 U.S. birth cohorts. Men

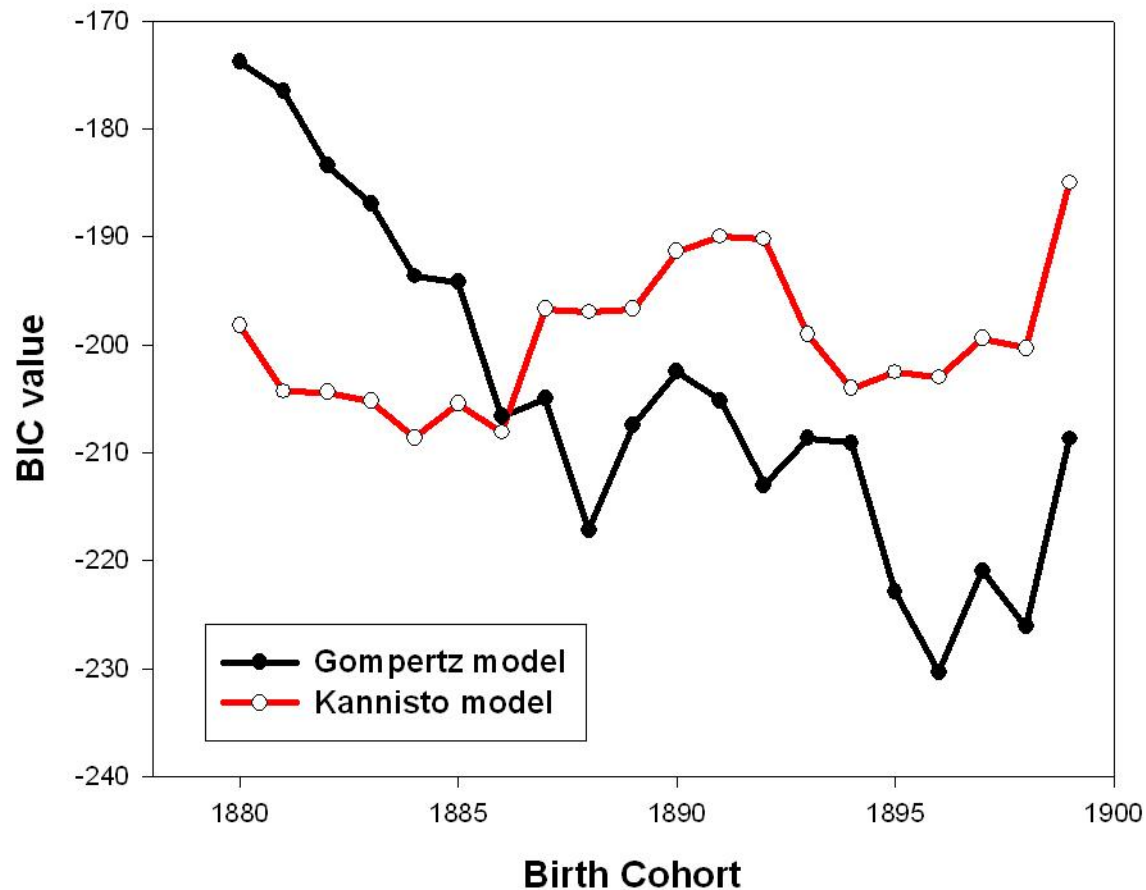
BIC values for fitting Gompertz and Kannisto models



Fitting age-specific cohort death rates taken from the Human Mortality Database

# 1880-1899 U.S. birth cohorts. Women

## BIC values for fitting Gompertz and Kannisto models



Fitting age-specific cohort death rates taken from the Human Mortality Database

# Conclusion

**Mortality deceleration is more prevalent in historically older birth cohorts when age reporting was less accurate**



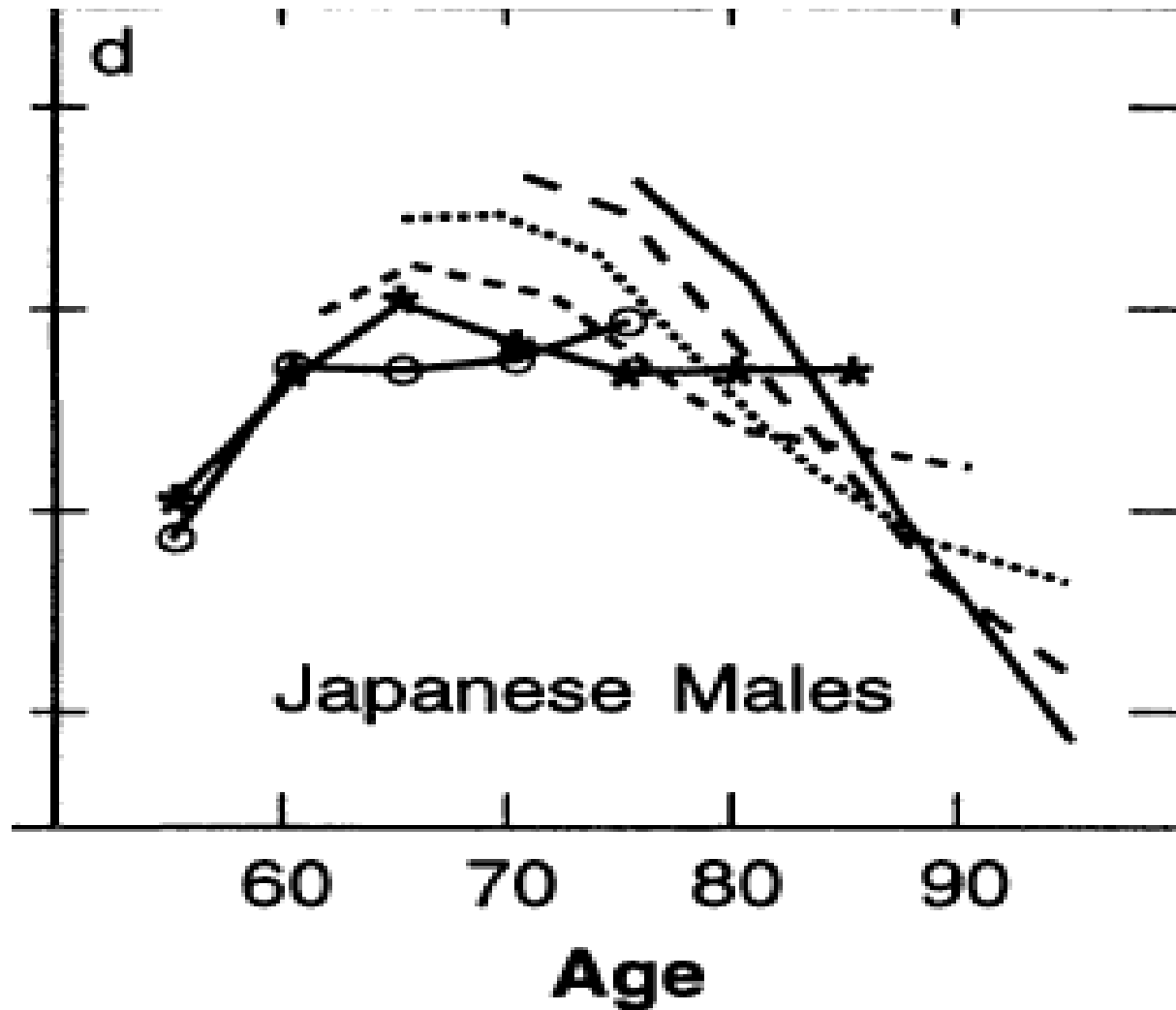
# **Alternative way to study mortality trajectories at advanced ages: Age-specific rate of mortality change**

**Suggested by Horiuchi and Coale (1990), Coale and Kisker (1990), Horiuchi and Wilmoth (1998) and later called 'life table aging rate (LAR)'**

$$k(x) = d \ln \mu(x) / dx$$

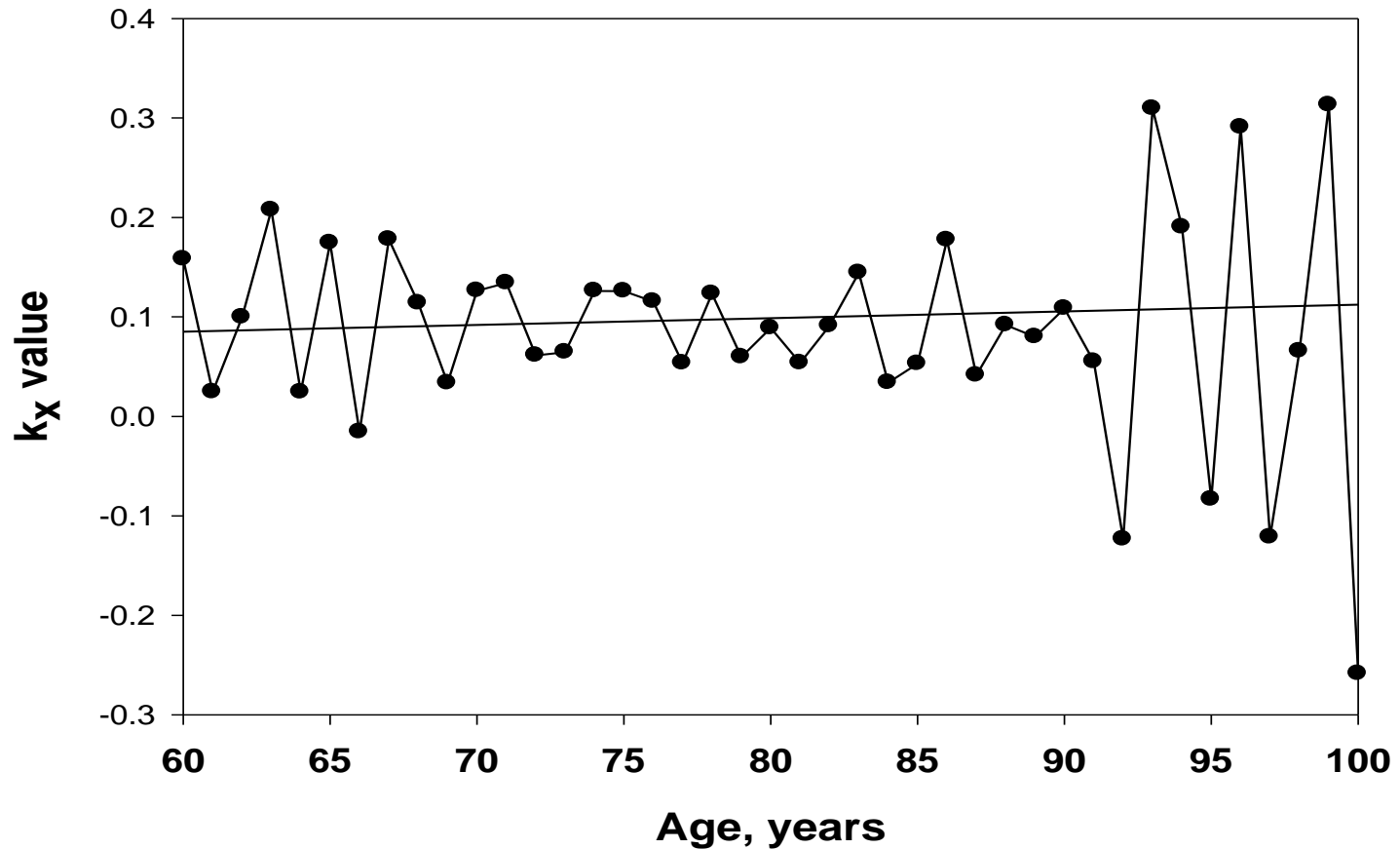
- **Constant  $k(x)$  suggests that mortality follows the Gompertz model.**
- **Earlier studies found that  $k(x)$  declines in the age interval 80-100 years suggesting mortality deceleration.**

# Typical result from Horiuchi and Wilmoth paper (Demography, 1998)



# Life-Table Aging Rate ( $k_x$ )

## Swedish males, 1896 birth cohort



Flat  $k(x)$  suggests that mortality follows the Gompertz law

# Slope coefficients (with p-values) for linear regression models of LAR on age

| Country | Sex | Birth cohort |         |          |         |          |         |
|---------|-----|--------------|---------|----------|---------|----------|---------|
|         |     | 1894         |         | 1896     |         | 1898     |         |
|         |     | slope        | p-value | slope    | p-value | slope    | p-value |
| Canada  | F   | -0.00023     | 0.914   | 0.00004  | 0.984   | 0.00066  | 0.583   |
|         | M   | 0.00112      | 0.778   | 0.00235  | 0.499   | 0.00109  | 0.678   |
| France  | F   | -0.00070     | 0.681   | -0.00179 | 0.169   | -0.00165 | 0.181   |
|         | M   | 0.00035      | 0.907   | -0.00048 | 0.808   | 0.00207  | 0.369   |
| Sweden  | F   | 0.00060      | 0.879   | -0.00357 | 0.240   | -0.00044 | 0.857   |
|         | M   | 0.00191      | 0.742   | -0.00253 | 0.635   | 0.00165  | 0.792   |
| USA     | F   | 0.00016      | 0.884   | 0.00009  | 0.918   | 0.000006 | 0.994   |
|         | M   | 0.00006      | 0.965   | 0.00007  | 0.946   | 0.00048  | 0.610   |

All regressions were run in the age interval 80-100 years.

# **Linear regression models of LAR on age for 1880-1899 U.S. birth cohorts**

Slope coefficients are negative and statistically significant for 1882, 1885 and 1886 male U.S. birth cohorts and 1880, 1881 and 1886 female birth cohorts suggesting mortality deceleration

Slope coefficients are statistically non-significant for 1880, 1881, 1883, 1884, 1887-1899 male U.S. birth cohorts and 1882-1885 and 1887-1899 female birth cohorts suggesting the Gompertz model

Conclusion: Mortality deceleration is prevalent only in historically old birth cohorts.

# Can data aggregation result in mortality deceleration?

Age-specific **5-year** cohort death rates taken from the Human Mortality Database

Studied countries: Canada, France, Sweden, United States

Studied birth cohorts: 1880-84, 1885-89, 1895-99

$k(x)$  calculated in the age interval 80-100 years

$k(x)$  calculated using one-year (age) mortality rates

# Slope coefficients (with p-values) for linear regression models of $k(x)$ on age

| Country | Sex | Birth cohort |              |          |              |          |              |
|---------|-----|--------------|--------------|----------|--------------|----------|--------------|
|         |     | 1885-89      |              | 1890-94  |              | 1895-99  |              |
|         |     | slope        | p-value      | slope    | p-value      | slope    | p-value      |
| Canada  | F   | -0.00069     | 0.372        | 0.00015  | 0.851        | -0.00002 | 0.983        |
|         | M   | -0.00065     | 0.642        | 0.00094  | 0.306        | 0.00022  | 0.850        |
| France  | F   | -0.00273     | <b>0.047</b> | -0.00191 | <b>0.005</b> | -0.00165 | <b>0.002</b> |
|         | M   | -0.00082     | 0.515        | -0.00049 | 0.661        | -0.00047 | 0.412        |
| Sweden  | F   | -0.00036     | 0.749        | -0.00122 | 0.185        | -0.00210 | 0.122        |
|         | M   | -0.00234     | 0.309        | -0.00127 | 0.330        | -0.00089 | 0.696        |
| USA     | F   | -0.00030     | 0.654        | -0.00027 | 0.685        | 0.00004  | 0.915        |
|         | M   | -0.00050     | 0.417        | -0.00039 | 0.399        | 0.00002  | 0.972        |

All regressions were run in the age interval 80-100 years.

# Conclusions

**Age-specific rate of mortality change remains flat (does not decrease) in the age interval 80-100 years for 24 studied single-year birth cohorts of Canada, France, Sweden and the United States suggesting that mortality follows the Gompertz law**

**Data aggregation may increase a tendency of spurious mortality slow down at advanced ages**



# **Mortality of Supercentenarians: Does It Grow with Age?**

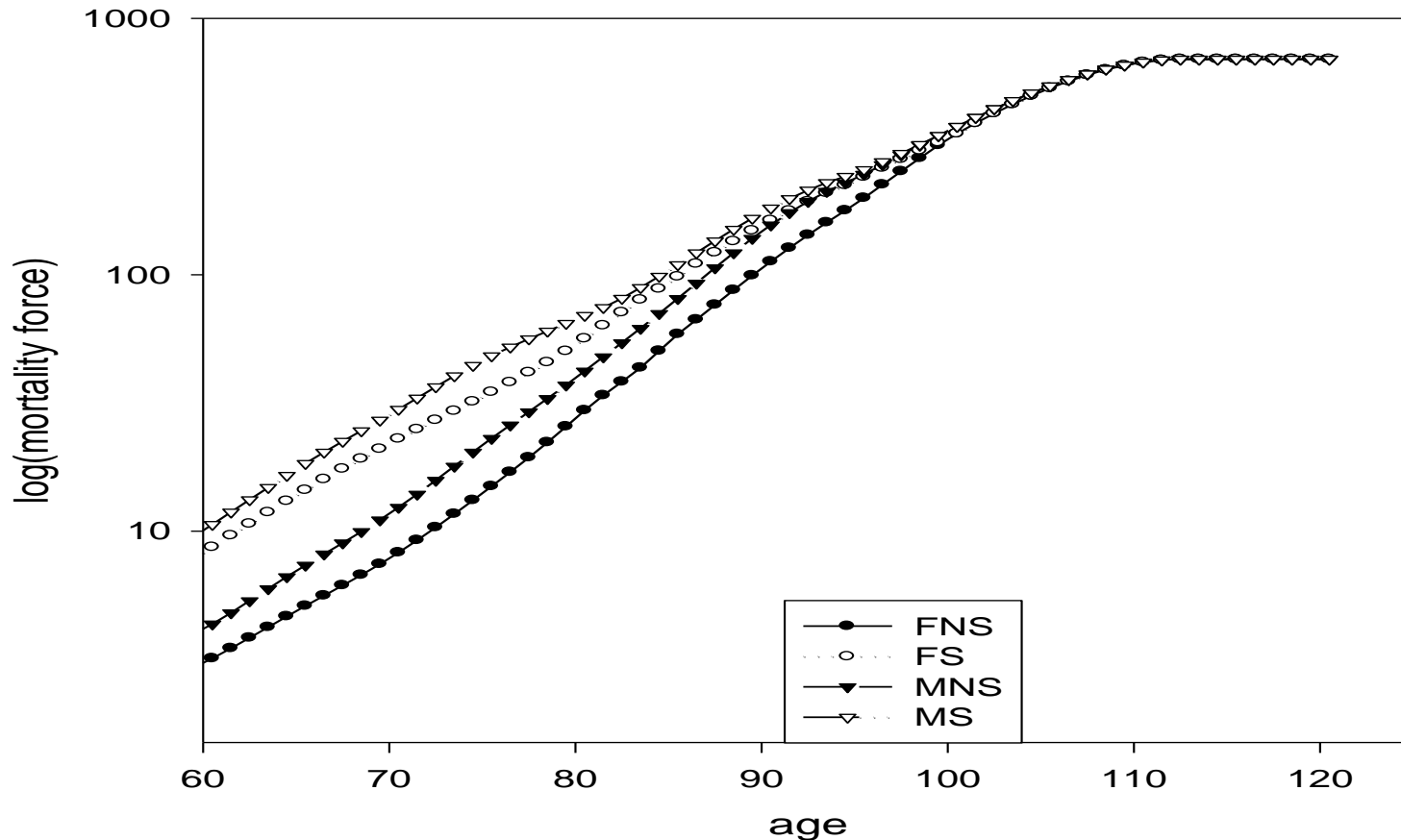
# Biodemography of human ageing

James W. Vaupel<sup>1,2,3</sup>

Most reported cases of a person being a centenarian — and to an even greater extent a supercentenarian — are erroneous<sup>73,74</sup>. To verify reputed high ages, correct birth records have to be found. A meticulous research endeavour has yielded a remarkable finding: between the validated ages of 110 and 114, the annual probability of death is constant at a level of 50% per year<sup>73</sup>. The sparse observations of survival after age 114 are not inconsistent with the hypothesis that mortality stays at this level at all ages after 110. As explained in Box 1, this result implies that at least at advanced ages, human individuals deteriorate at the same rate.

# Actuarial 2014 valuation basic tables (VBT) suggest flat mortality after age 110 years

VBT 2014



MNS, FNS – male and female nonsmokers, MS, FS - smokers

# **International Database on Longevity (IDL)**

**This database contains validated records of persons aged 110 years and more from 15 countries with good quality of vital records.**

**The contributors to IDL performed data collection in a way that avoided age-ascertainment bias, which is essential for demographic analysis.**

**The database was last updated in March 2010.**

**Available at [www.supercentenarians.org](http://www.supercentenarians.org)**

# **Previous studies of mortality using IDL**

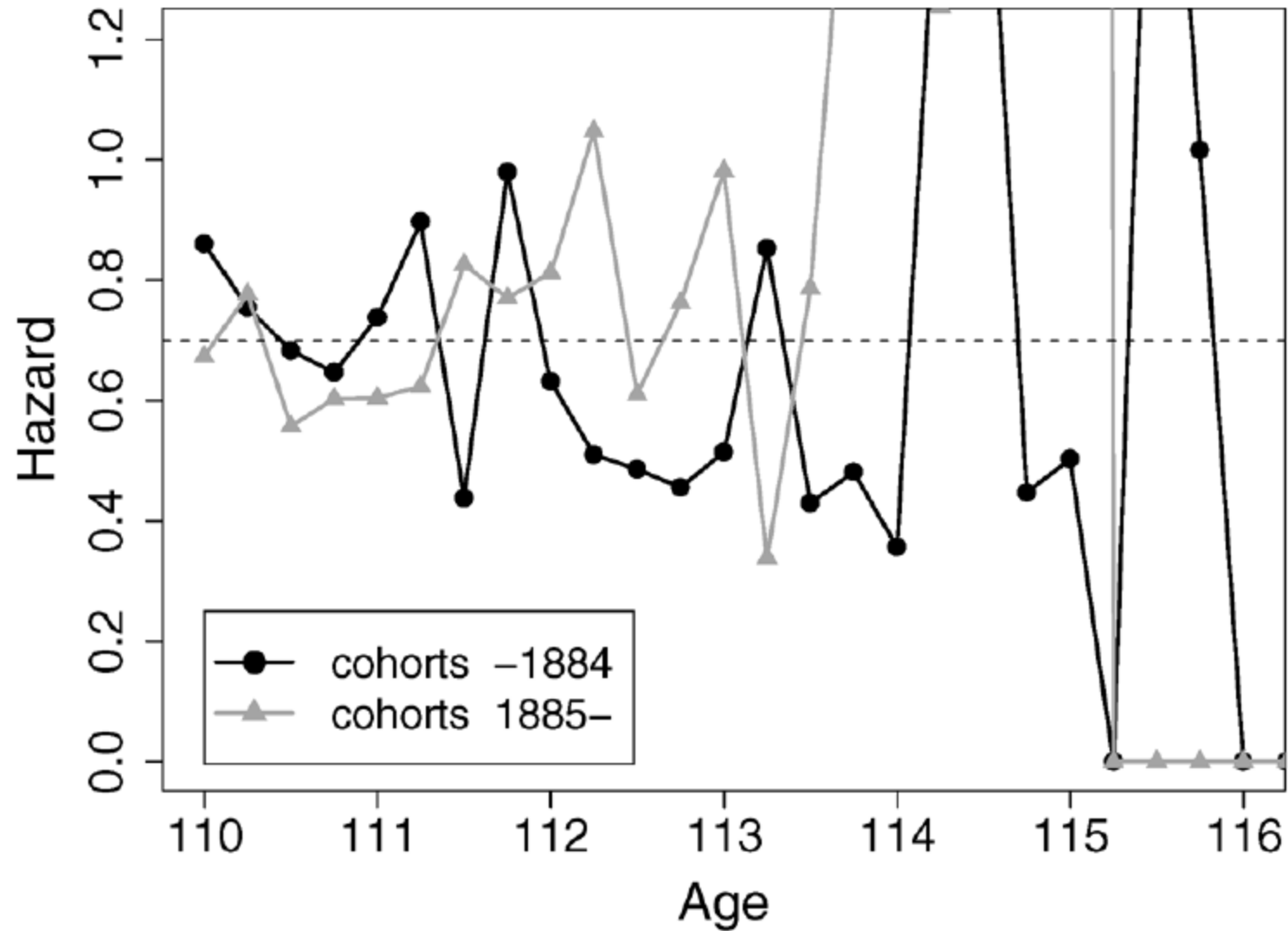
**Robine and Vaupel, 2001.**

**Robine et al. (2005). Used IDL data, calculated age-specific probabilities of death.**

**Gampe, 2010. Used IDL data. Wrote her own program to estimate hazard rates, which adjusts for censored and truncated data.**

**Main conclusion from these studies is that hazard rate after age 110 years is flat.**

# From study by Gampe (2010)



# **Our study of supercentenarians based on IDL data**

**IDL database as of January, 2015. Last update in 2010, last deaths in 2007.**

**Two extinct birth cohorts (<1885 and 1885-1892), so no censored or truncated records were used.**

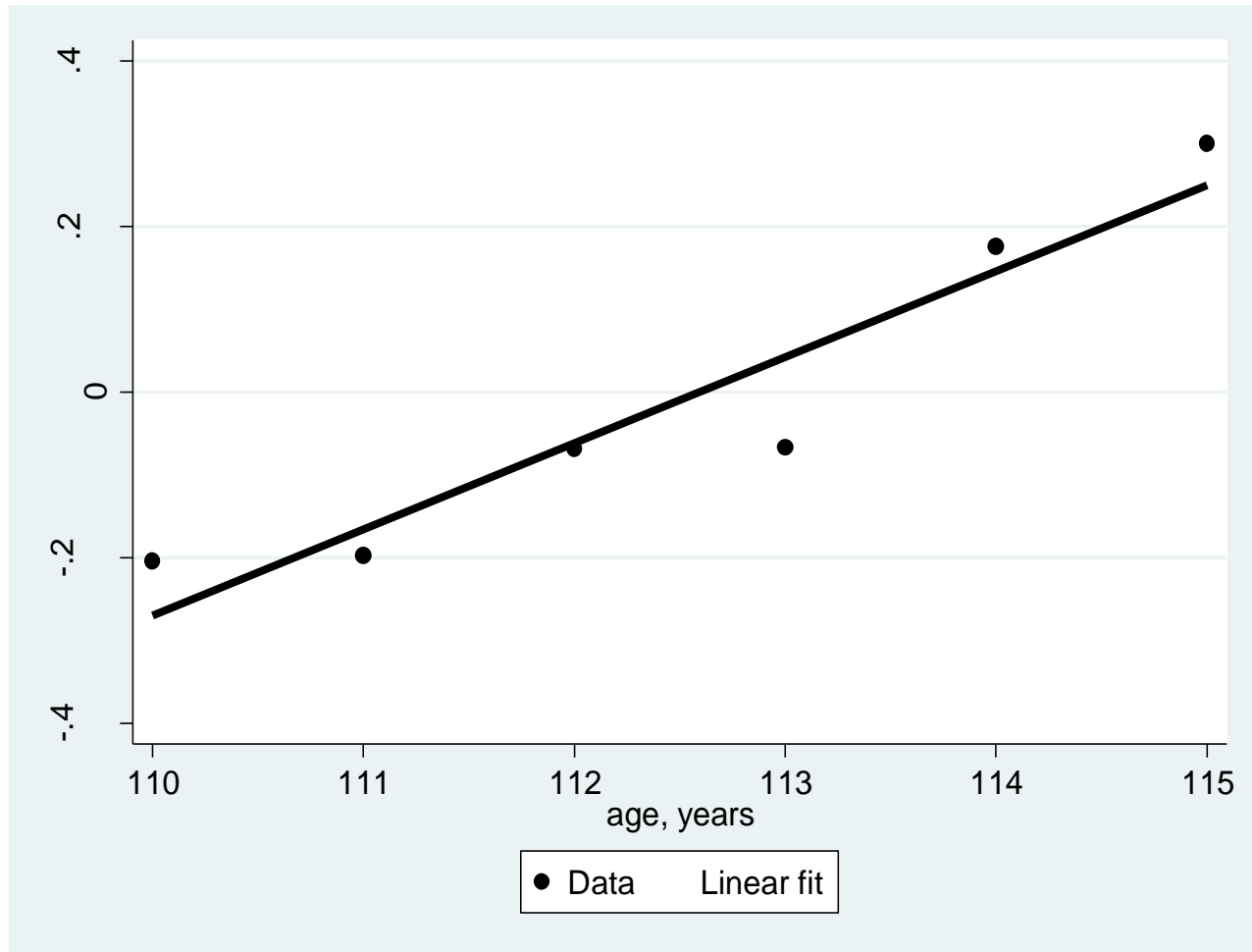
**Hazard rate was estimated using standard Stata package (procedure ltable).**

**Hazard rate was calculated using actuarial estimate of hazard rate (mortality rate):**

$$\mu_{x + \frac{\Delta x}{2}} = \frac{2}{\Delta x} \frac{l_x - l_{x + \Delta x}}{l_x + l_{x + \Delta x}}$$

# Mortality of supercentenarians

## Cohort born in 1885-1892

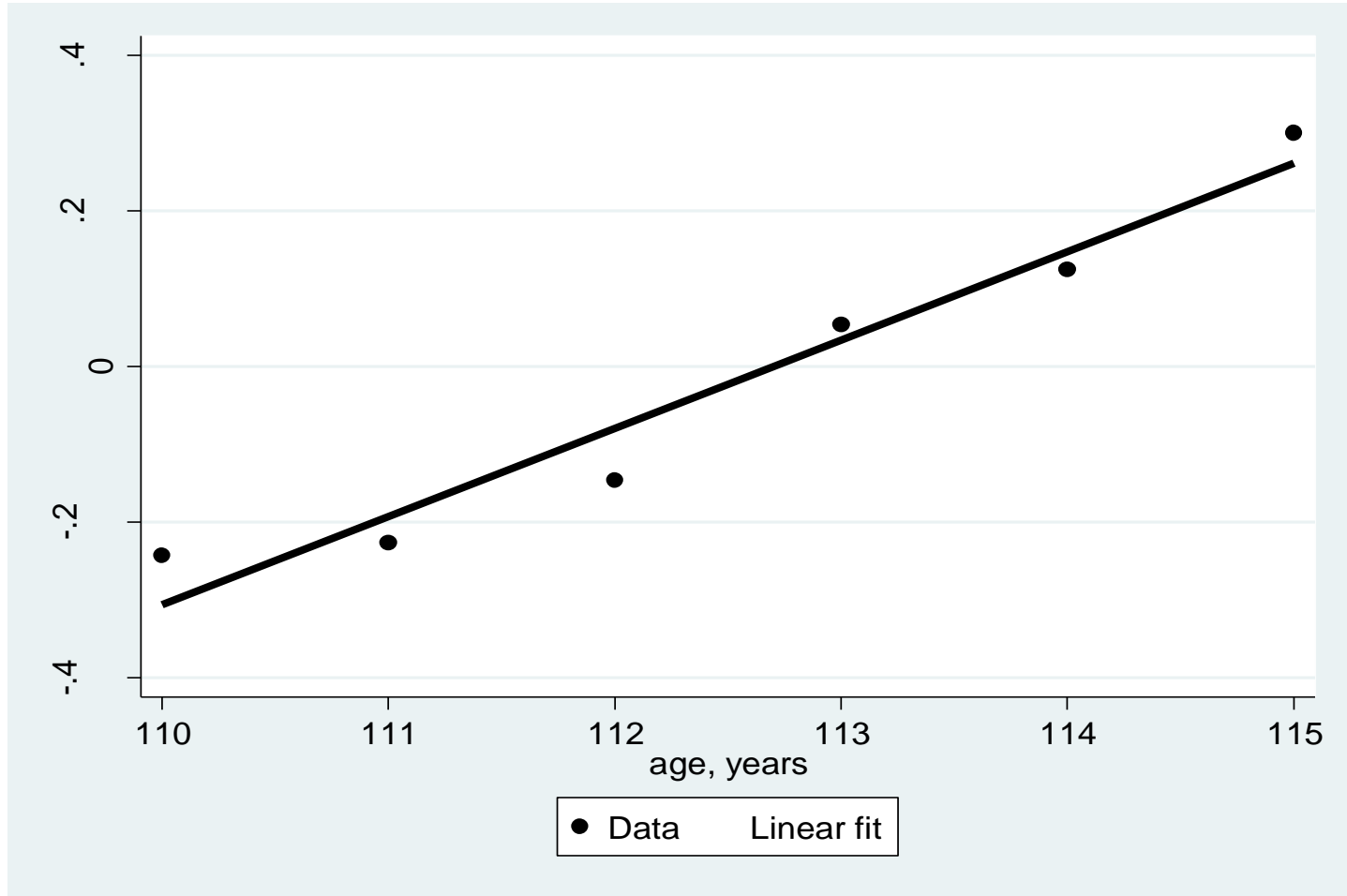


Yearly age intervals



# Mortality of supercentenarians

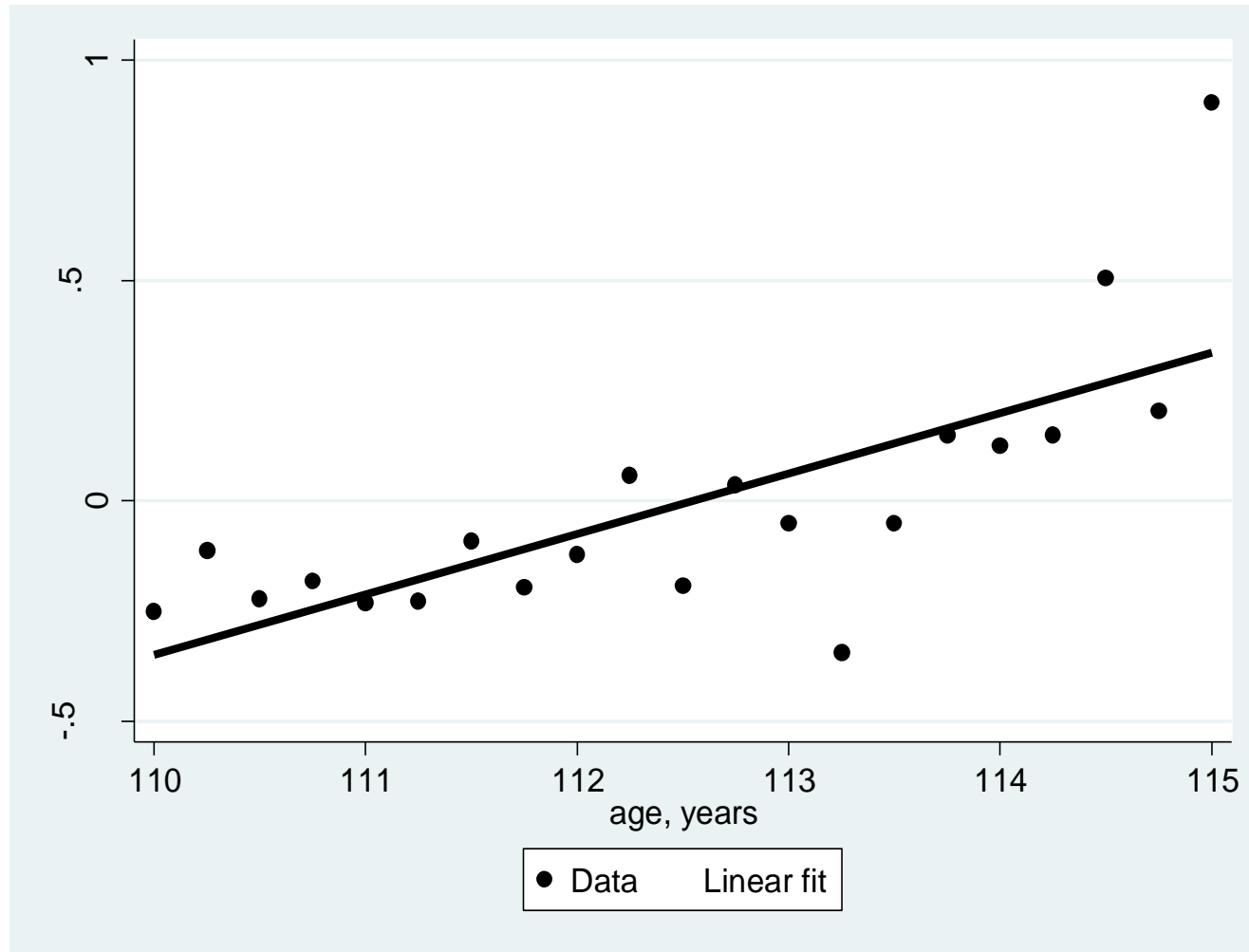
## U.S. cohort born in 1885-1892



Yearly age intervals

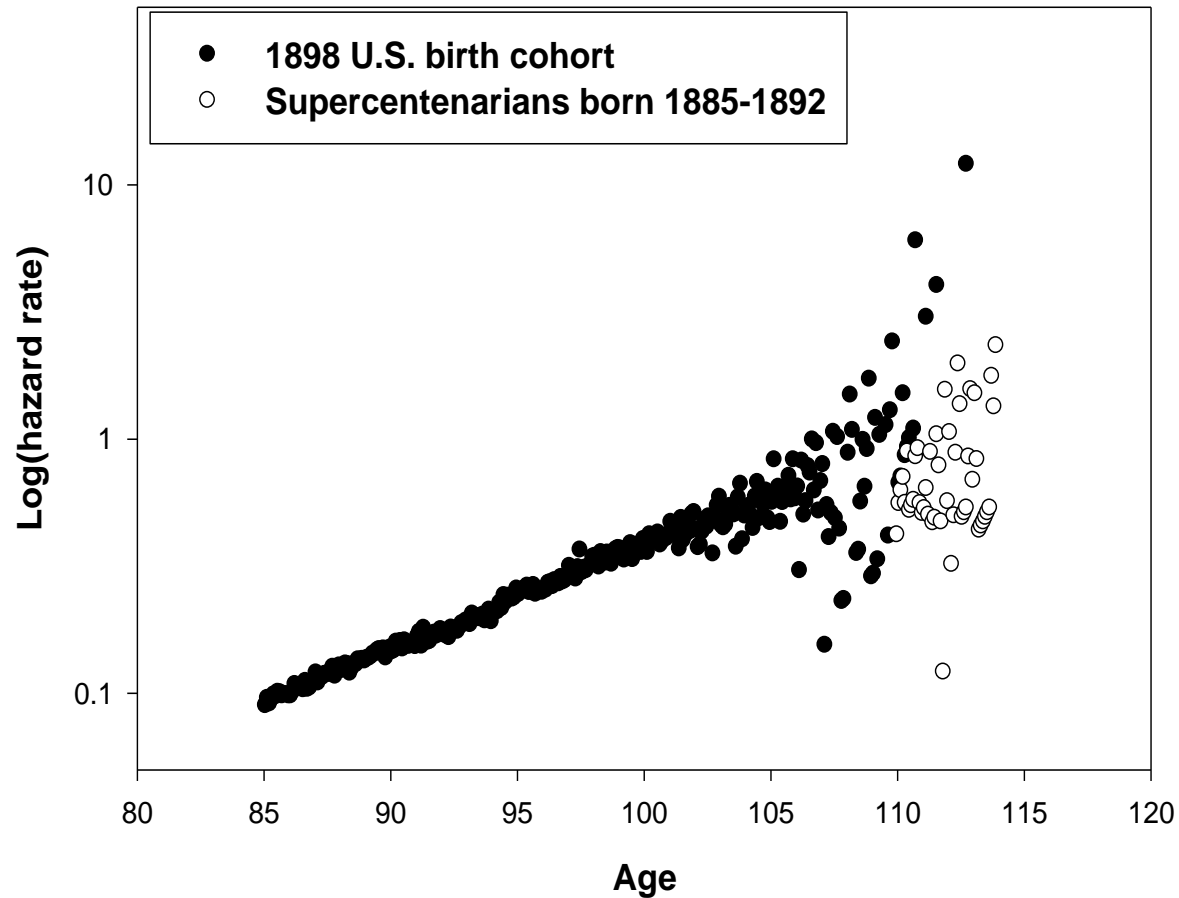
# Mortality of supercentenarians

## Cohort born in 1885-1892



Quarterly age intervals

# Mortality after age 85 years



Monthly age intervals. Data for 1898 U.S. birth cohort are taken from the SSA DMF

# Testing assumption about flat hazard rate after age 110

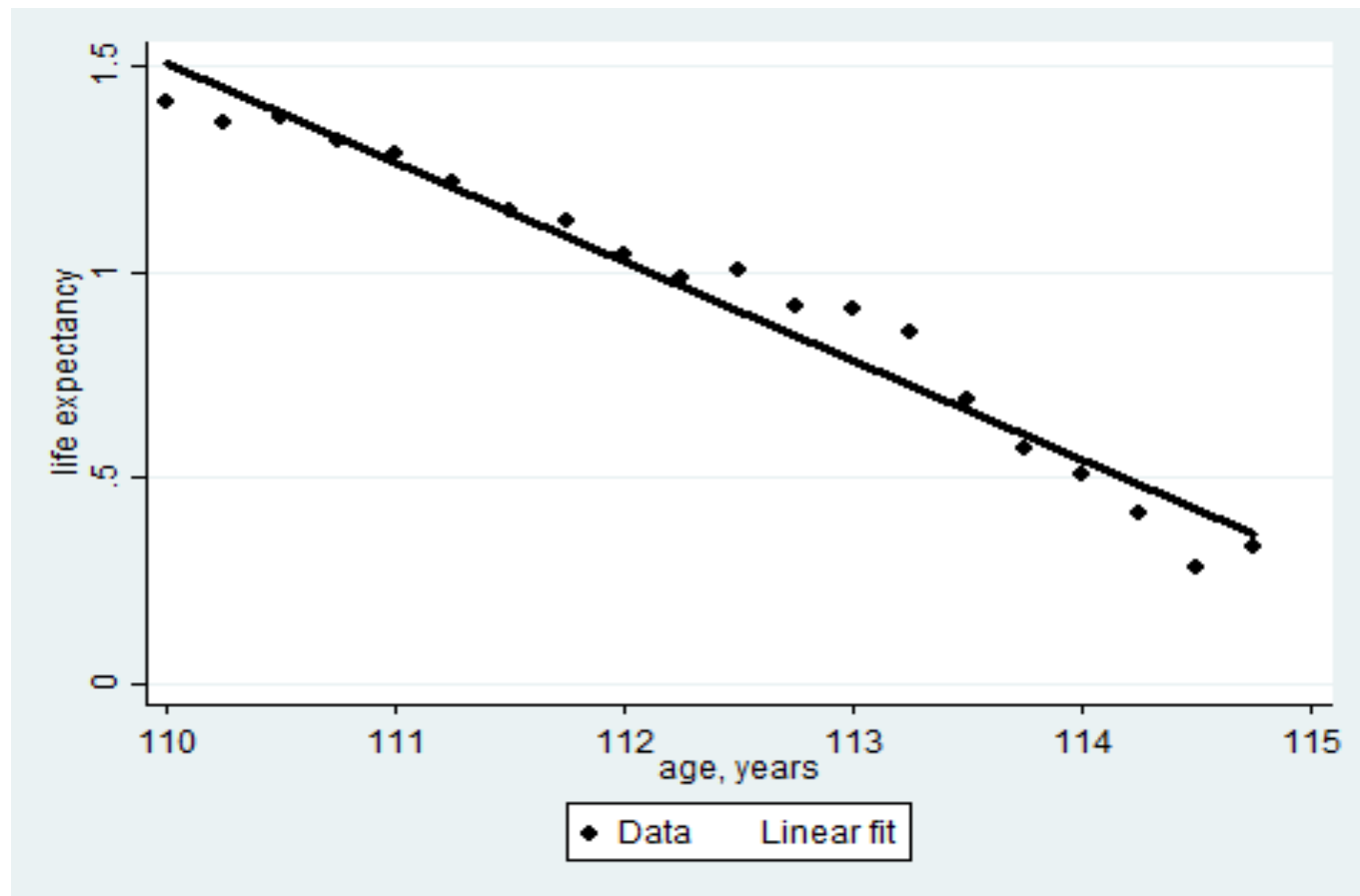
Direct estimates of hazard rates at advanced ages are subjected to huge variations.

More robust ways of testing this assumption come from the properties of exponential distribution:

1. Hazard rate,  $\mu = \text{const}$
2. Mean life expectancy (LE) =  $1/\mu = \text{const}$
3. Coefficient of variation for LE =  $\text{SD}/\text{mean}=1$

# Mean remaining life expectancy vs age

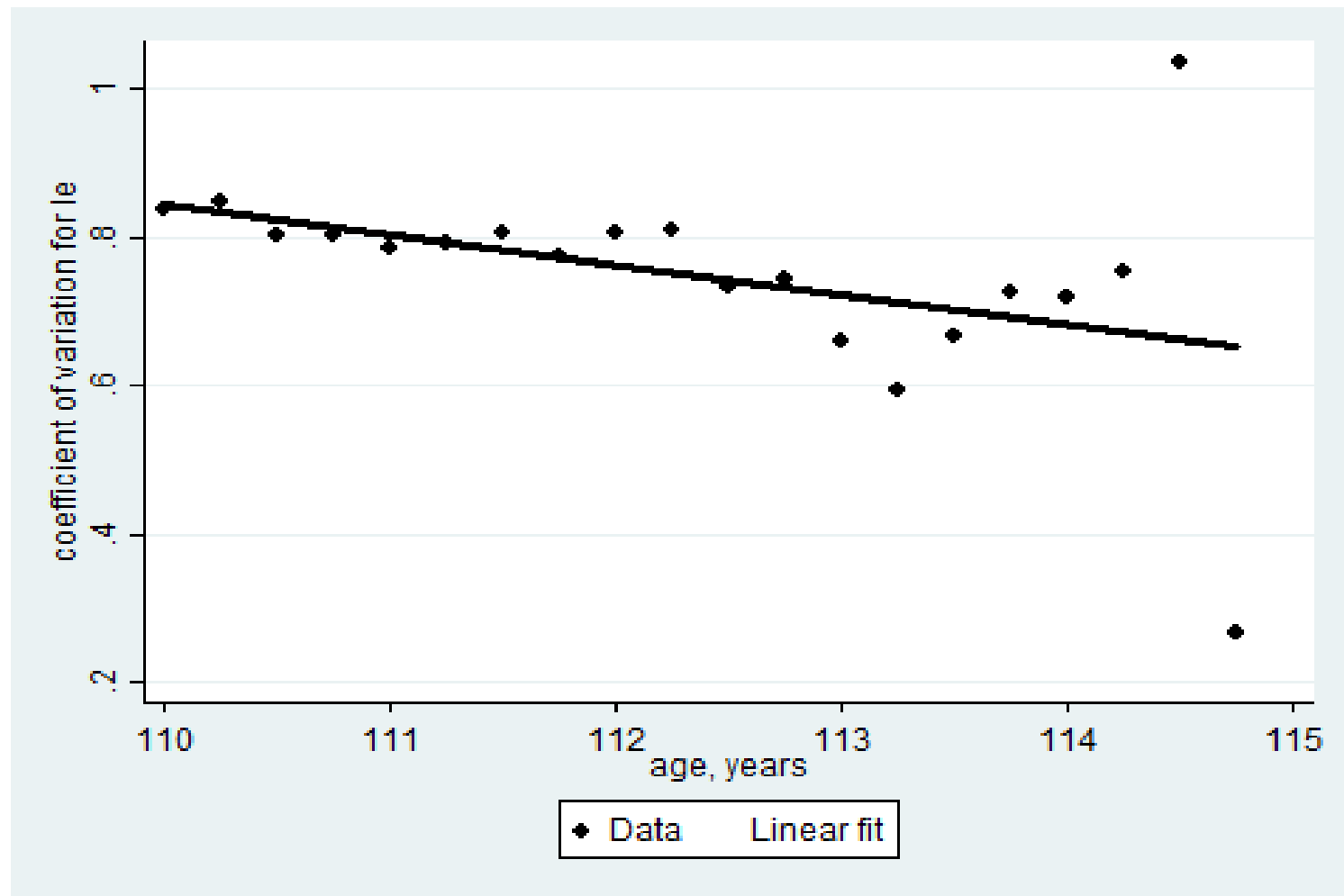
## Cohort born in 1885-1892



Slope coefficient = -0.24 ( $p < 0.001$ ). Quarterly age intervals

# Coefficient of variation for LE vs age

## Cohort born in 1885-1892



Slope coefficient = -0.041 (p=0.066). Quarterly age intervals

# Conclusions

**Assumption about flat hazard rate after age 110 years is not supported by the study of age trajectory for mean life expectancy. Life expectancy after age 110 is declining suggesting that actuarial aging continues.**

**Coefficient of variation for LE is lower than one and declines rather than increases with age, which does not support the assumption about flat hazard rate.**

**Hazard rates (mortality rates) after age 110 continue to grow with almost linear trajectory in semi-log coordinates suggesting that Gompertz law is still working**

# **Which estimate of hazard rate is the most accurate?**

**Simulation study comparing several existing estimates:**

- **Nelson-Aalen estimate available in Stata**
- **Sacher estimate (Sacher, 1956)**
- **Simplified Sacher estimate (Gehan, 1969)**
- **Actuarial estimate (Kimball, 1960)**



# Simulation study to identify the most accurate mortality indicator

Simulate yearly  $l_x$  numbers assuming Gompertz function for hazard rate in the entire age interval and initial cohort size equal to  $10^{11}$  individuals

Gompertz parameters are typical for the U.S. birth cohorts: slope coefficient (alpha) =  $0.08 \text{ year}^{-1}$ ;  
 $R_0 = 0.0001 \text{ year}^{-1}$

Focus on ages beyond 90 years

Accuracy of various hazard rate estimates (Sacher, Gehan, and actuarial estimates) and probability of death is compared at ages 100-110

# Simulation study of Gompertz mortality

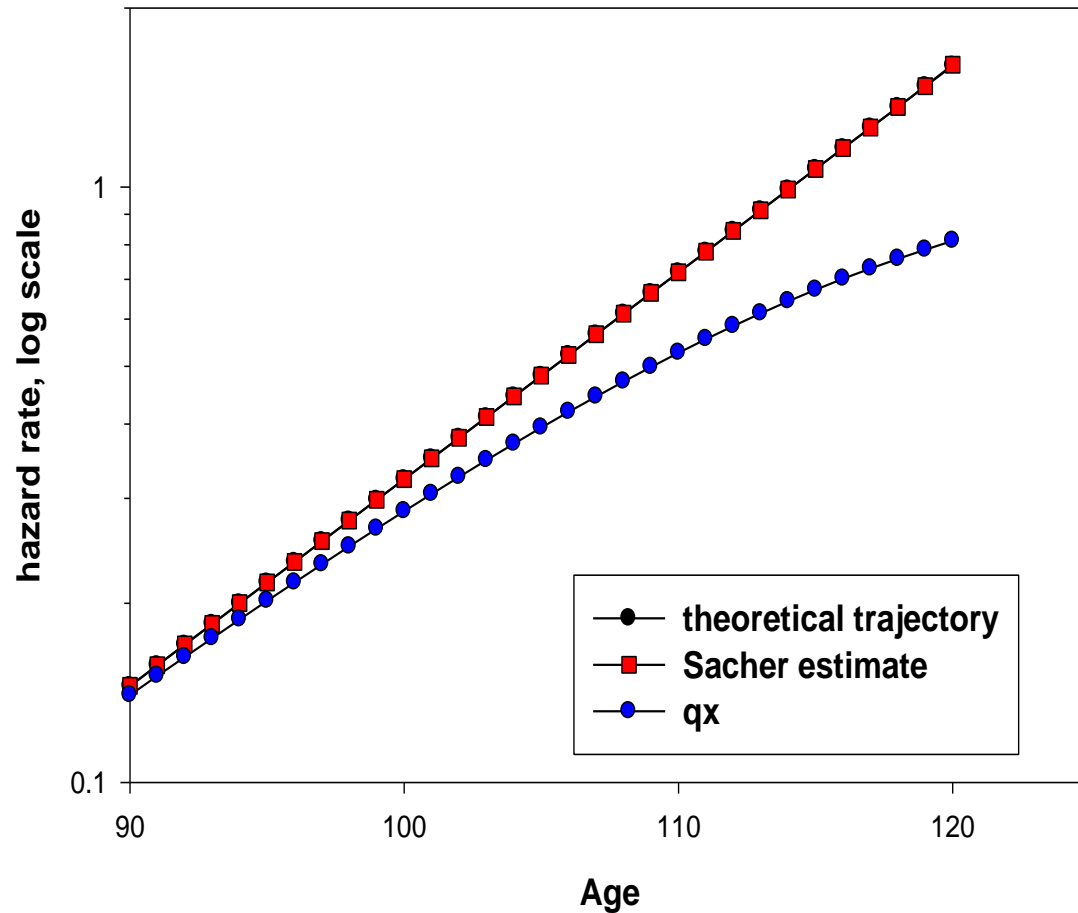
## Compare Sacher hazard rate estimate and probability of dying in a yearly age interval

Sacher estimates practically coincide with theoretical mortality trajectory

$$\mu_x = \frac{1}{2\Delta x} \ln \frac{l_{x-\Delta x}}{l_{x+\Delta x}}$$

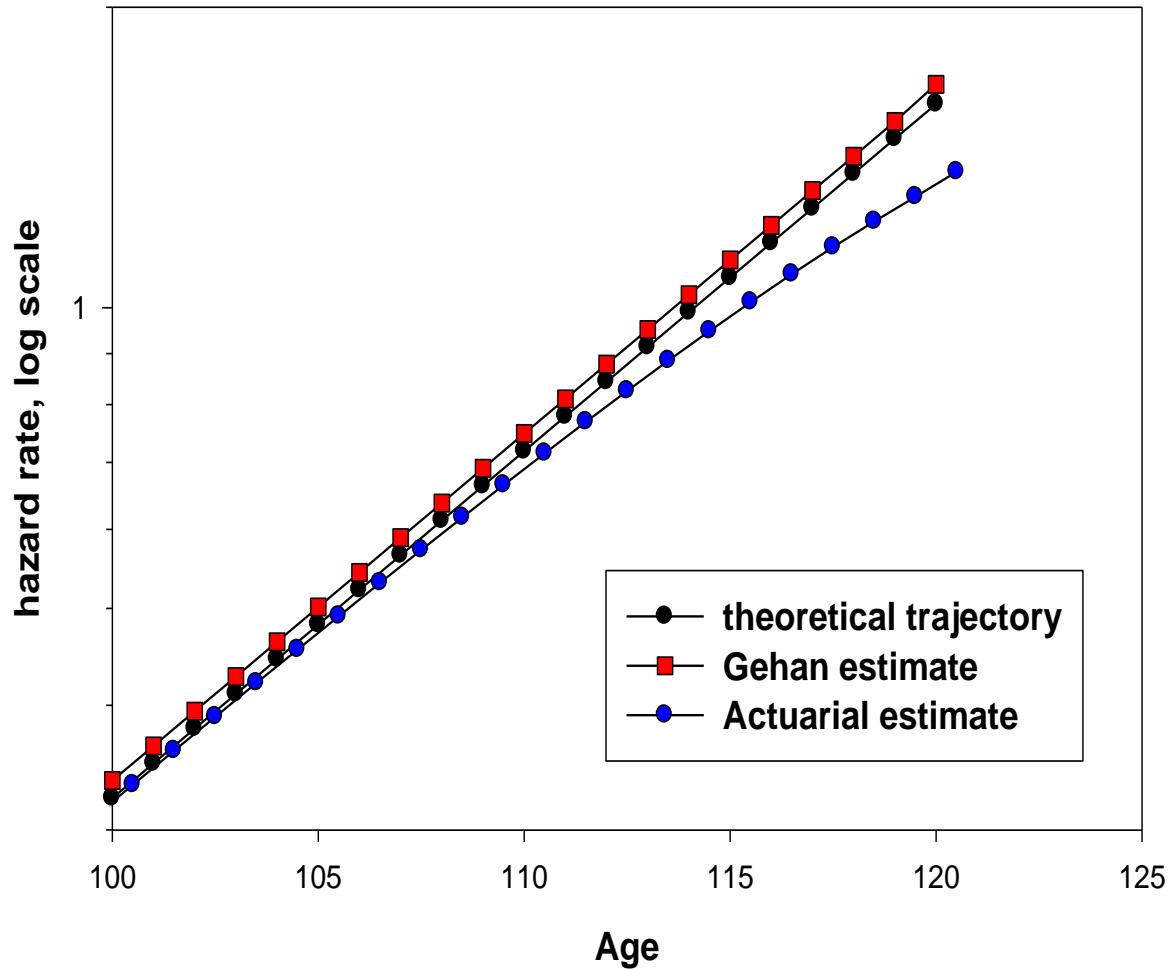
Probability of dying values strongly underestimate mortality after age 100

$$q_x = \frac{d_x}{l_x}$$



# Simulation study of Gompertz mortality

## Compare Gehan and actuarial hazard rate estimates



Simplified Sacher estimates slightly overestimate hazard rate because of its half-year shift to earlier ages

$$\mu_x = -\ln(1 - q_x)$$

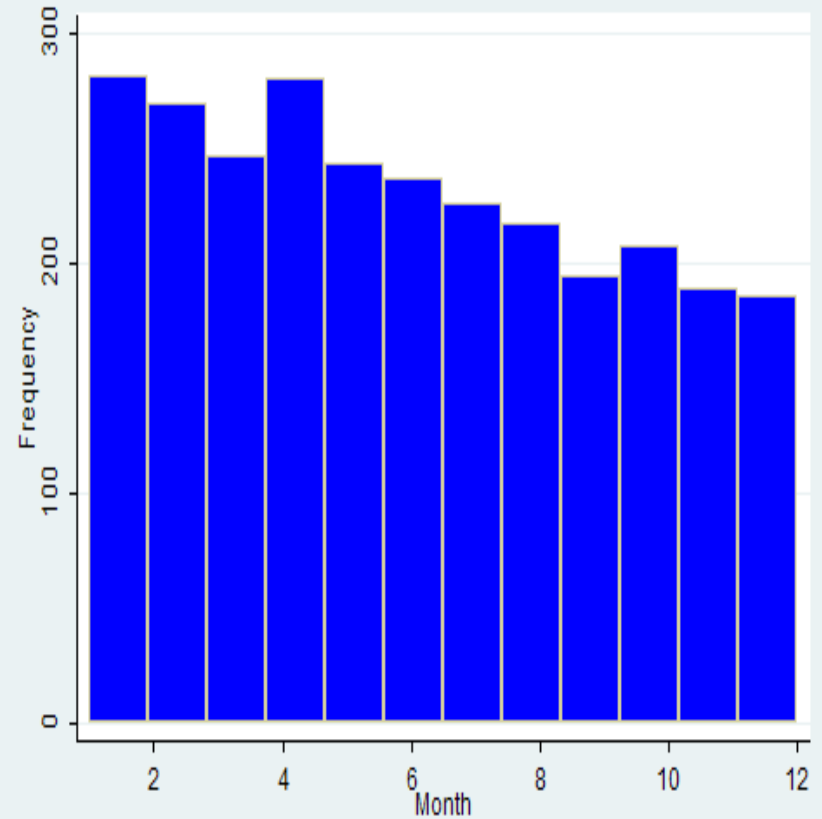
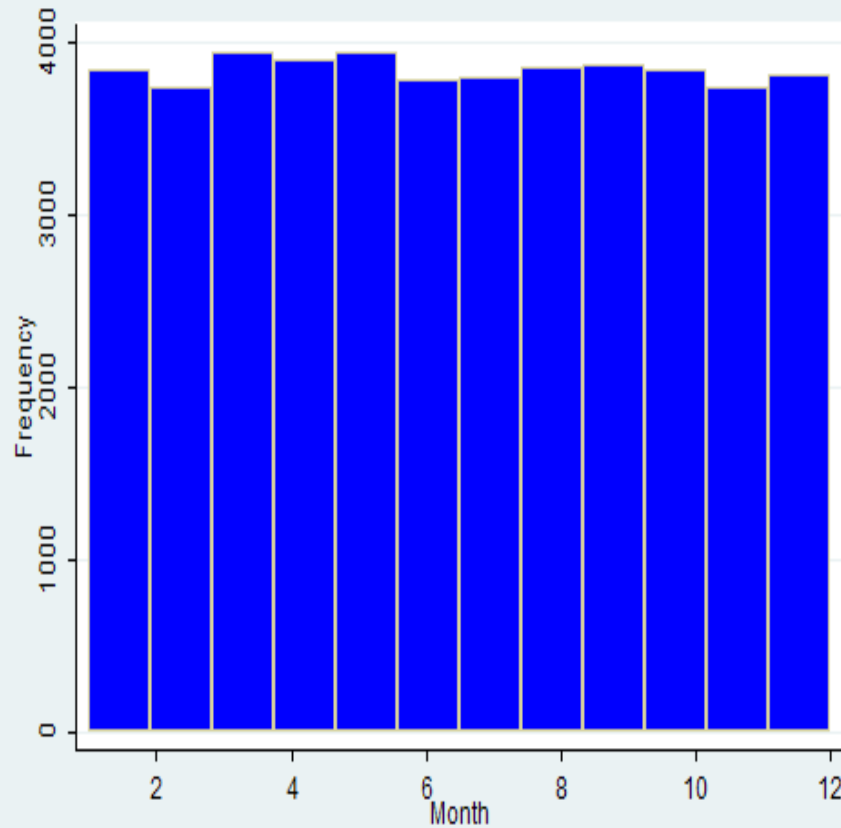
Actuarial estimates (death rates) underestimate mortality after age 100

$$\mu_{x + \frac{\Delta x}{2}} = \frac{2}{\Delta x} \frac{l_x - l_{x + \Delta x}}{l_x + l_{x + \Delta x}}$$

# Deaths at extreme ages are not distributed uniformly over one-year interval

85-year olds

102-year olds



1894 birth cohort from the Social Security Death Index

# Accuracy of hazard rate estimates

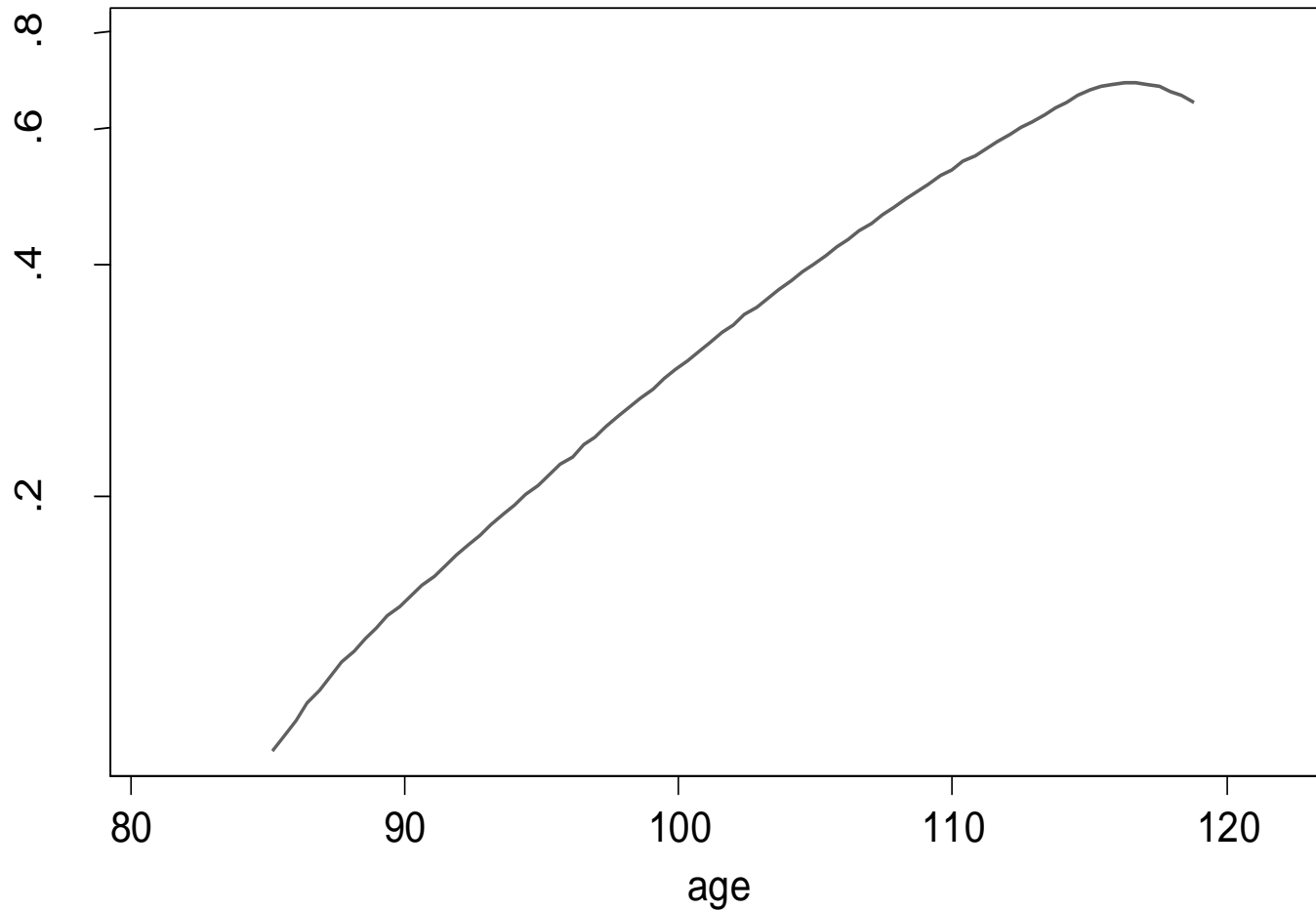
Relative difference between theoretical and observed values, %

| Estimate             | 100 years         | 110 years         |
|----------------------|-------------------|-------------------|
| Probability of death | 11.6%, understate | 26.7%, understate |
| Sacher estimate      | 0.1%, overstate   | 0.1%, overstate   |
| Gehan estimate       | 4.1%, overstate   | 4.1%, overstate   |
| Actuarial estimate   | 1.0%, understate  | 4.5%, understate  |
|                      |                   |                   |

# Simulation study of the Gompertz mortality

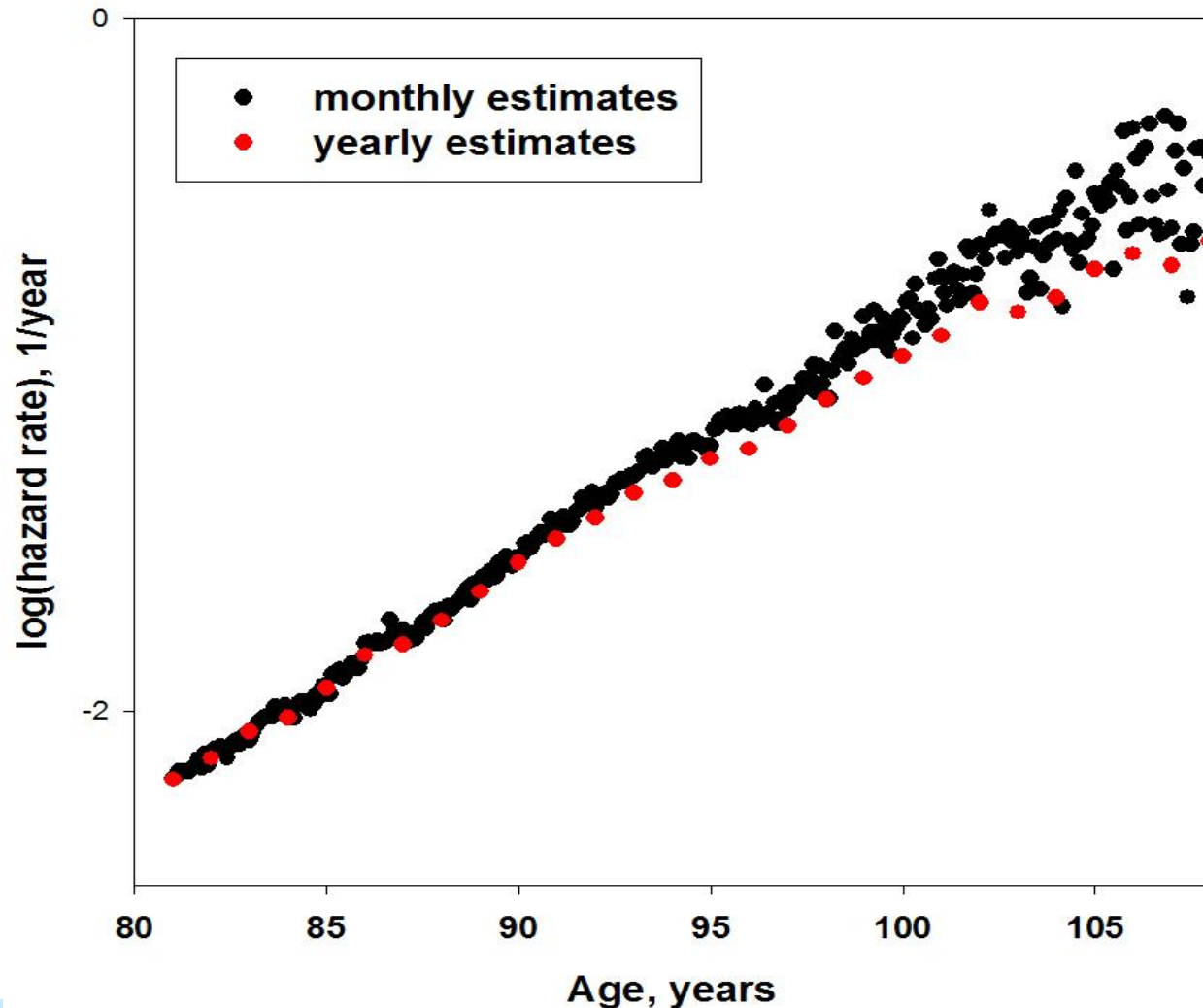
## Kernel smoothing of hazard rates

Smoothed hazard estimate



# Mortality of 1894 birth cohort

## Monthly and Yearly Estimates of Hazard Rates using Nelson-Aalen formula (Stata)



# Sacher formula for hazard rate estimation (Sacher, 1956; 1966)

$$\mu_x = \frac{1}{\Delta x} \left( \ln l_{x - \frac{\Delta x}{2}} - \ln l_{x + \frac{\Delta x}{2}} \right) = \frac{1}{2\Delta x} \ln \frac{l_{x - \Delta x}}{l_{x + \Delta x}}$$

Hazard rate

$l_x$  - survivor function at age  $x$ ;  $\Delta x$  - age interval

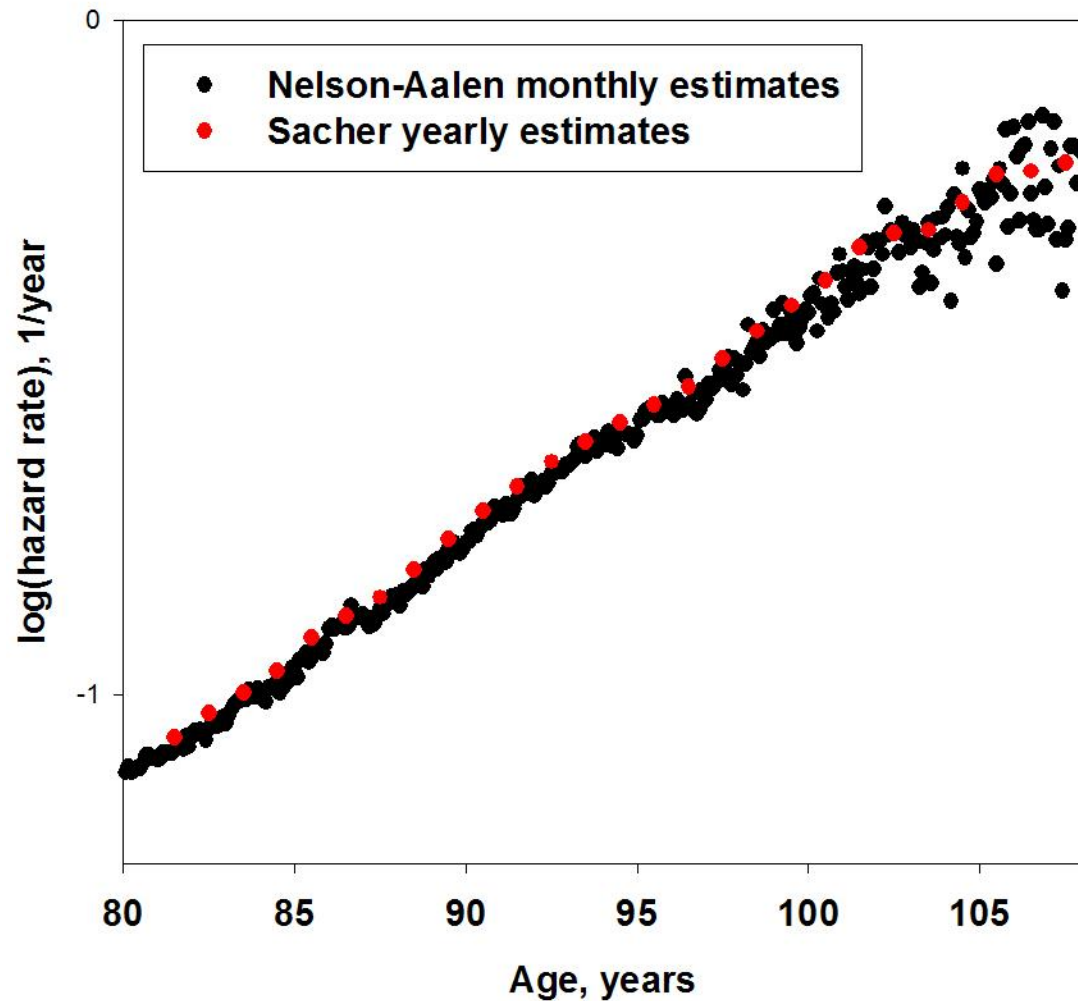
Simplified version suggested by Gehan (1969):

$$\mu_x = -\ln(1-q_x)$$



# Mortality of 1894 birth cohort

## Sacher formula for yearly estimates of hazard rates



# Conclusions

**Deceleration of mortality in later life is more expressed for data with lower quality.**

**Quality of age reporting in DMF becomes poor beyond the age of 107 years**

**Below age 107 years and for data of reasonably good quality the Gompertz model fits mortality better than the logistic model (no mortality deceleration)**

**Sacher estimate of hazard rate turns out to be the most accurate and most useful estimate to study mortality at advanced ages**

# Acknowledgments

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Center on Aging, NORC/University of Chicago

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**What about mortality  
deceleration in other species?**

# Mortality Deceleration in Other Species

## **Invertebrates:**

**Nematodes, shrimps, bdelloid rotifers, degenerate medusae (Economos, 1979)**

**Drosophila melanogaster (Economos, 1979; Curtsinger et al., 1992)**

**Medfly (Carey et al., 1992)**

**Housefly, blowfly (Gavrillov, 1980)**

**Fruit flies, parasitoid wasp (Vaupel et al., 1998)**

**Bruchid beetle (Tatar et al., 1993)**

## **Mammals:**

**Mice (Lindop, 1961; Sacher, 1966; Economos, 1979)**

**Rats (Sacher, 1966)**

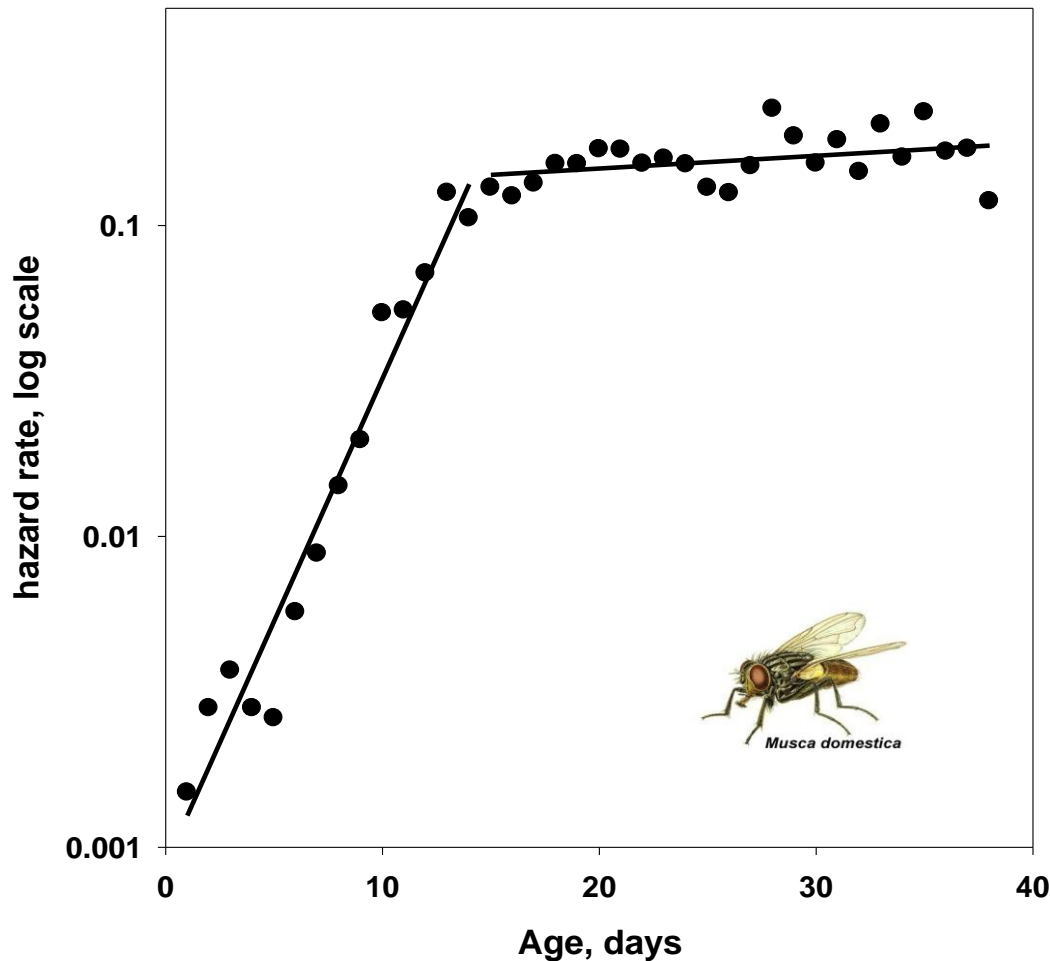
**Horse, Sheep, Guinea pig (Economos, 1979; 1980)**

**However no mortality deceleration is reported for**

**Rodents (Austad, 2001)**

**Baboons (Bronikowski et al., 2002)**

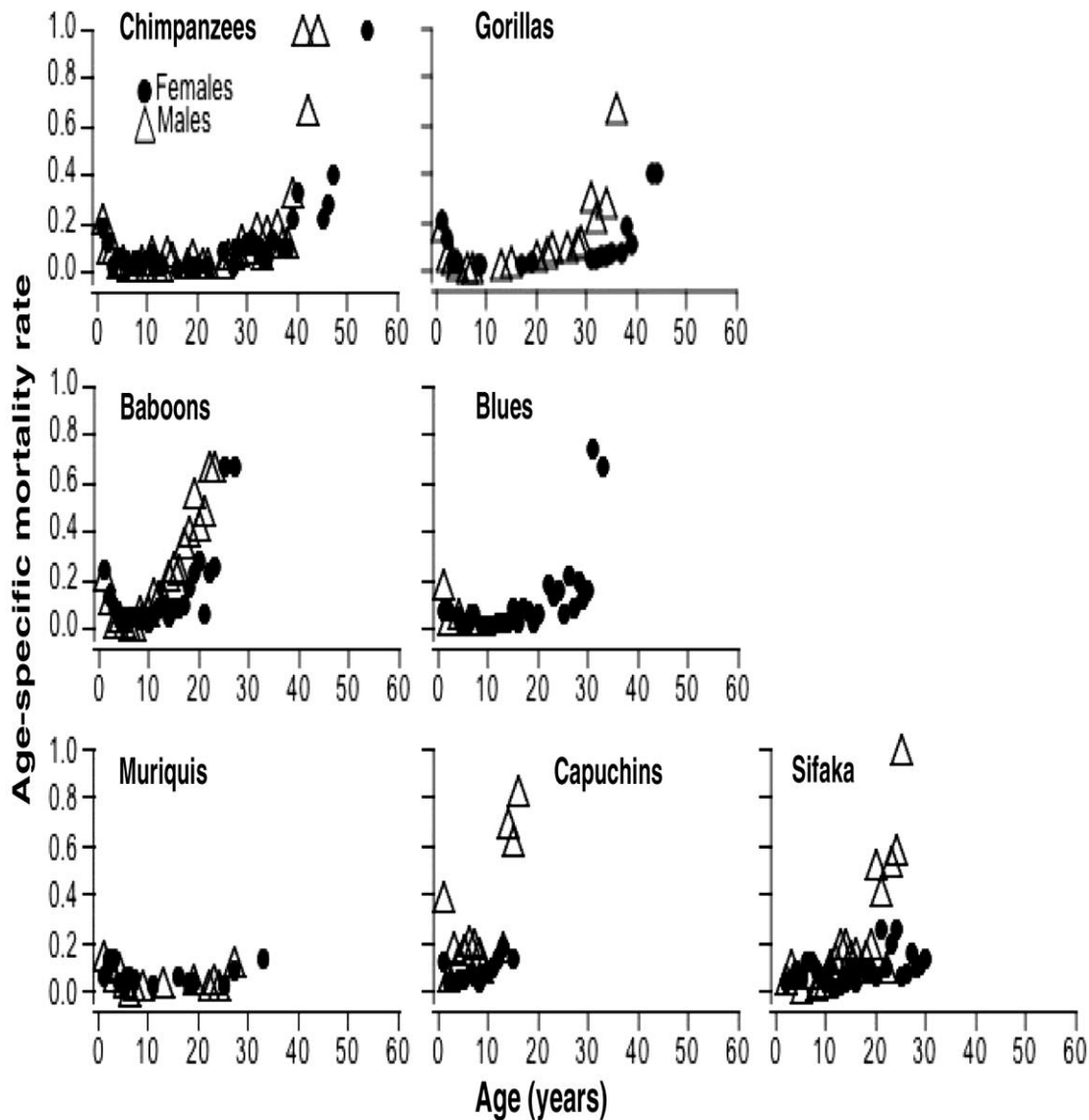
# Mortality Leveling-Off in House Fly *Musca domestica*



Based on life table of 4,650 male house flies published by Rockstein & Lieberman, 1959



# Recent developments



“none of the age-specific mortality relationships in our nonhuman primate analyses demonstrated the type of leveling off that has been shown in human and fly data sets”

**Bronikowski et al.,  
Science, 2011**



# What about other mammals?



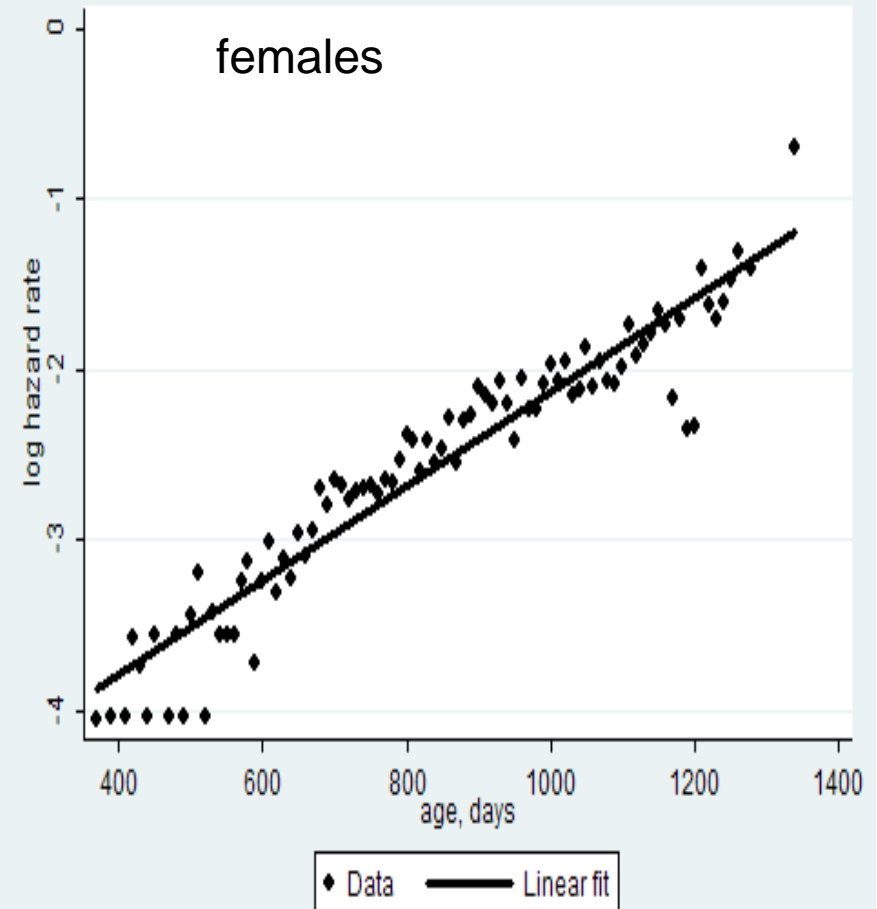
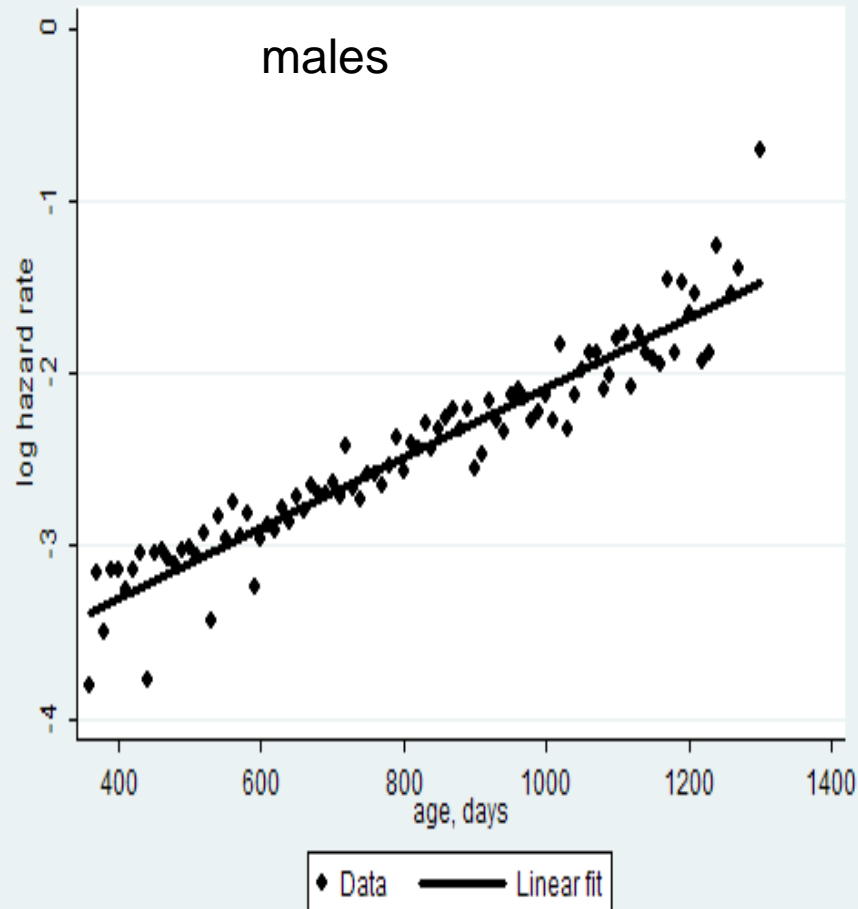
## **Mortality data for mice:**

**Data from the NIH Interventions Testing Program,  
courtesy of Richard Miller (U of Michigan)**

**Argonne National Laboratory data, courtesy  
of Bruce Carnes (U of Oklahoma)**

# Mortality of mice (log scale)

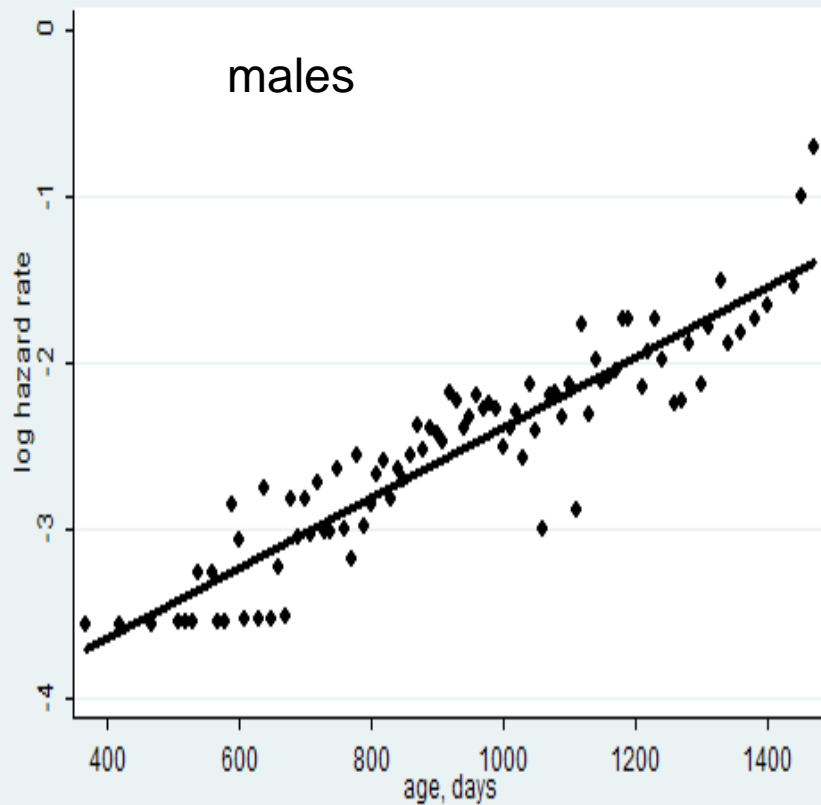
## Miller data



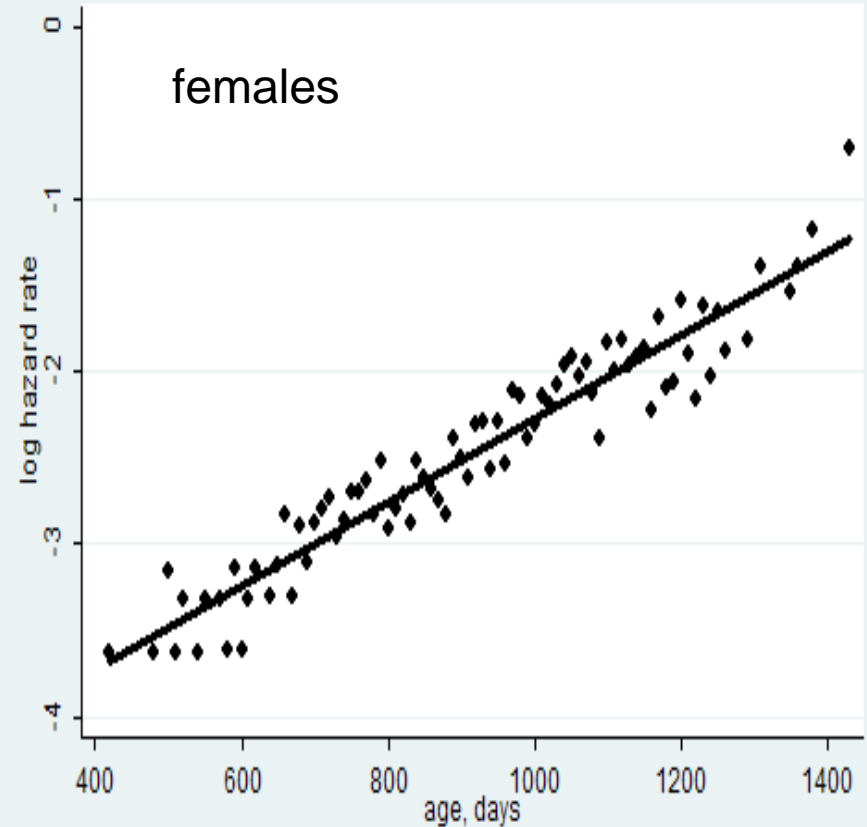
**Actuarial estimate of hazard rate with 10-day age intervals**

# Mortality of mice (log scale)

## Carnes data



◆ Data — Linear fit



◆ Data — Linear fit

**Actuarial estimate of hazard rate with 10-day age intervals**

**Data were collected by the Argonne National Laboratory, early experiments shown**

# Bayesian information criterion (BIC) to compare the Gompertz and logistic models, mice data

| Dataset                     | Miller data Controls |        | Miller data Exp., no life extension |        | Carnes data Early controls |        | Carnes data Late controls |        |
|-----------------------------|----------------------|--------|-------------------------------------|--------|----------------------------|--------|---------------------------|--------|
|                             | M                    | F      | M                                   | F      | M                          | F      | M                         | F      |
| Sex                         |                      |        |                                     |        |                            |        |                           |        |
| Cohort size at age one year | 1281                 | 1104   | 2181                                | 1911   | 364                        | 431    | 487                       | 510    |
| Gompertz                    | -597.5               | -496.4 | -660.4                              | -580.6 | -585.0                     | -566.3 | -639.5                    | -549.6 |
| logistic                    | -565.6               | -495.4 | -571.3                              | -577.2 | -556.3                     | -558.4 | -638.7                    | -548.0 |

Better fit (lower BIC) is highlighted in red

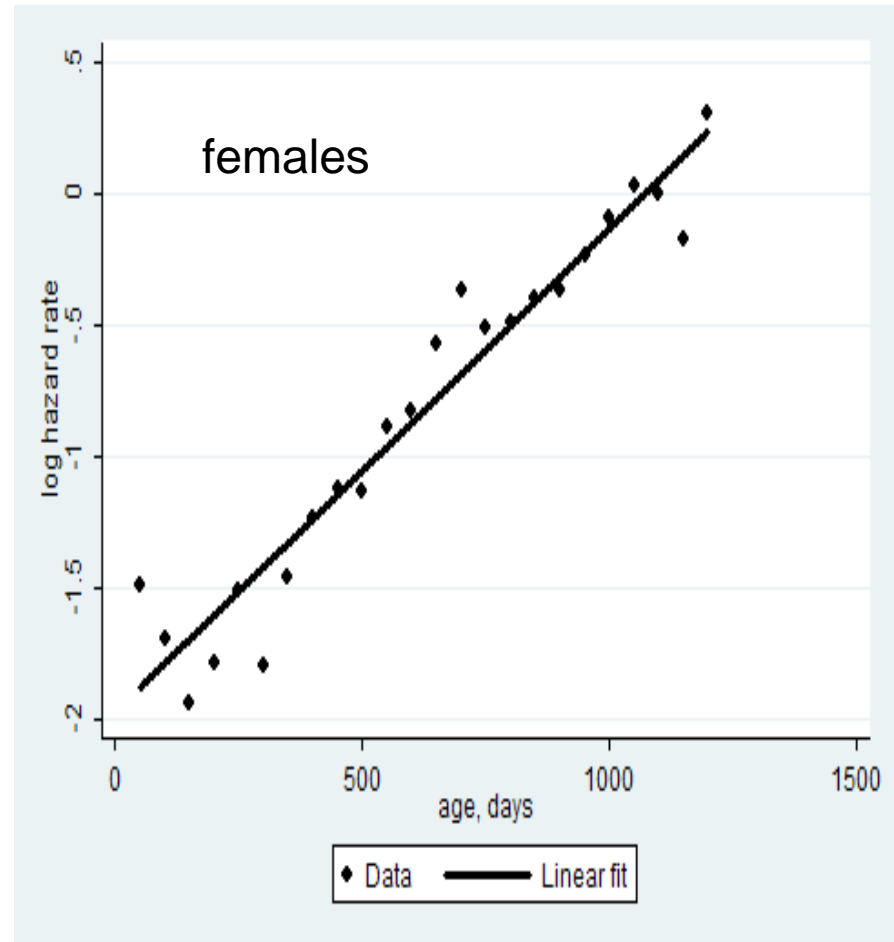
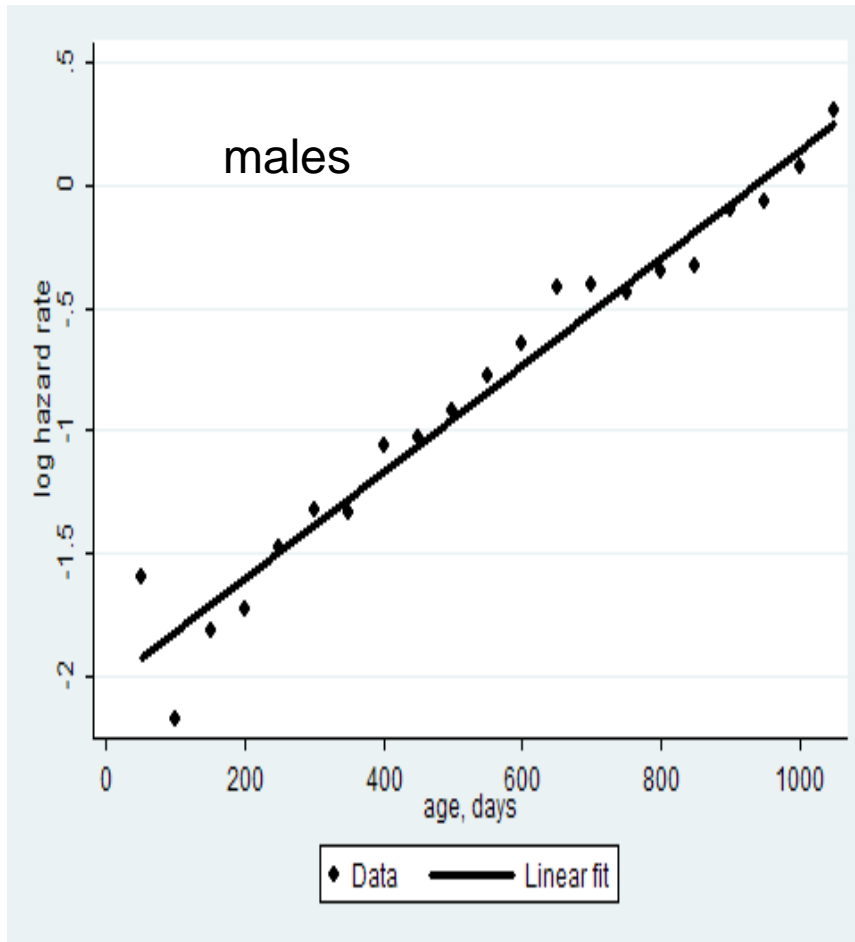
**Conclusion: In all cases Gompertz model demonstrates better fit than logistic model for mortality of mice after one year of age**

# Laboratory rats



**Data sources: Dunning, Curtis (1946);  
Weisner, Sheard (1935), Schlettwein-Gsell  
(1970)**

# Mortality of Wistar rats



**Actuarial estimate of hazard rate with 50-day age intervals**

**Data source: Weisner, Sheard, 1935**

# Bayesian information criterion (BIC) to compare logistic and Gompertz models, rat data

| Line        | Wistar (1935) |       | Wistar (1970) |       | Copenhagen |       | Fisher |       | Backcrosses |       |
|-------------|---------------|-------|---------------|-------|------------|-------|--------|-------|-------------|-------|
|             | M             | F     | M             | F     | M          | F     | M      | F     | M           | F     |
| Sex         |               |       |               |       |            |       |        |       |             |       |
| Cohort size | 1372          | 1407  | 1372          | 2035  | 1328       | 1474  | 1076   | 2030  | 585         | 672   |
| Gompertz    | -34.3         | -10.9 | -34.3         | -53.7 | -11.8      | -46.3 | -17.0  | -13.5 | -18.4       | -38.6 |
| logistic    | 7.5           | 5.6   | 7.5           | 1.6   | 2.3        | -3.7  | 6.9    | 9.4   | 2.48        | -2.75 |

Better fit (lower BIC) is highlighted in red

**Conclusion: In all cases Gompertz model demonstrates better fit than logistic model for mortality of laboratory rats**

# Some other recent studies

*Scandinavian Actuarial Journal*, 2014

Vol. 2014, No. 3, 189–207, <http://dx.doi.org/10.1080/03461238.2012.676562>



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## Original Article

### **Beyond the Gompertz law: exploring the late-life mortality deceleration phenomenon**

MARK BEBBINGTON<sup>a\*</sup>, REBECCA GREEN<sup>a</sup>, CHIN-DIEW LAI<sup>a</sup> and  
RIČARDAS ZITIKIS<sup>b</sup>

A number of data sets have been explored, with a particular emphasis on those originating from Scandinavia. Although those from Australia, Canada, and the USA are compatible with Gompertzian mortality, those from the other countries examined are not. We find in particular that the onset of mortality deceleration is being progressively delayed in Western societies but that there is evidence of mortality plateauing at earlier ages.